Component Value-at-Risk to Identify Downside
Risk as Perceived by Shareholders

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JEL Codes: G3, G32, G1, G14

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Abstract
Multinational companies face increasing risks arising from external risk factors e.g. exchange rates, interest rates and commodity prices. These firms have responded by managing such risks through derivatives. However, reporting is still in its infancy and, hence, risks are less transparent to investors. We develop the ‘Component Value-at-Risk (VaR)’ framework for companies to identify downside risk as perceived by shareholders. This framework allows for decomposition into components attributable to the underlying risk factors. Perceived VaR should correspond, both in terms of composition and dynamics, to the real VaR as it is known by the company. Any differences potentially lead to surprises at times of earnings announcements and thus constitute a litigation threat to the firm.

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1 Introduction

An increasing number of companies is adopting value creation as a key corporate objective. The focus on profit exceeding cost of capital rather than profit per se is best understood by the surge of shareholder activism in the 1990s. Incorporating cost of capital in objectives and, hence, performance measures shows a company’s explicit attention for the compensation required by those who bear the risk of providing capital. An important prerequisite for establishing the appropriate compensation is risk transparency.

For multinational companies risk arising from external risk factors, e.g. exchange rates, interest rates and commodity prices, is increasing. The main reasons for such increase are, on the one hand, exposures that are rising because of globalising business and, on the other hand, increased volatility in the financial markets in the last few decades (Smithson (1998)). Firms have responded by embracing the concept of financial risk management. Neutralising excess risk through offsetting positions in derivatives is widespread corporate practice. Recent empirical research shows that 80 percent of large US companies and an even higher share of German firms use derivatives to manage risk ((Bodnar et al. (1998), Bodnar and Gebhardt (1999), Allayannis and Ofek (2001) and Howton and Perfect (1998)). Since current reporting in this field is still in its infancy, transparency of risk to investors is at stake. Any misperceptions potentially lead to surprises at times of earnings announcements and thus constitute a litigation threat. This is detrimental to a firm’s reputation in financial markets. Court rulings1 and a recent American Stock Exchange initiative demonstrate that shareholder concern is not just theoretical:

1 We refer to Smithson (1998, p. 493-496) for a detailed treatment of the Brane v. Roth and the Compaq Computer ruling. In both cases plaintiffs held shares in the company and claimed directors were liable for large losses incurred due to adverse price changes. In the first case, shareholders argued that the company should have hedged, whereas in the second they blamed the company for a lack of disclosure. In both cases the judge ruled in favour of the shareholders.
The objective of this study is to develop a tool that enables companies to assess shareholders’ perception of risk. We take the empirical evidence on corporate risk management as a starting point and therefore know that companies develop and apply internal risk measures. The company thus knows its true risk and the way it is built up from various sources of risk. To make sure that this risk is sufficiently transparent to financial markets the company needs to know how shareholders perceive its risk. It is the crux of this paper to develop a tool that measures perceived risk and the way it is built up from various sources of risk. Comparing true risk to perceived risk and the way both are built up enables the company to identify potential misperceptions at an early stage. The company can now correct for any potential misperception through a targeted communication effort. If, for example, the true risk emanating from a dollar Deutschmark exposure is larger than perceived by the market, the company can address its foreign exchange exposures and its hedging policy in a press release. We consider Value-at-Risk (VaR) an appropriate risk measure because it is a measure of downside risk. This is the most important element of risk since negative surprises increase litigation threat. Furthermore, it provides an easy to understand summary statistic. For a broad and accessible overview of VaR we refer to Duffie and Pan (1997). The use of VaR for corporates is a relatively unexplored field that offers many theoretical and empirical challenges.

Using insights from both the VaR and multifactor literature we develop the ‘Component VaR’ framework that defines and explores a company’s VaR in terms of market value at risk. Since shareprice dynamics and thus market value dynamics reflect changing shareholders’ perceptions, this VaR measures perceived risk. We add three innovative features. First, a dynamic operating environment, changing company operations and hedging policies are likely to affect a firm’s exposure to external risk factors over time. We deal with these by allowing for dynamic exposures.
Exposure estimates based on information until today are used to make a Component VaR forecast for the coming period. Second, the Component VaR is non-parametric and hence does not require restrictive distributional assumptions such as normality. Third, the Component VaR allows for decomposition of overall VaR into components attributable to the underlying external risk factors. This enables us to evaluate the contribution of each risk factor to the overall VaR. The decomposition result is a powerful, general result. It only builds on the linearity of the factor model and therefore applies irrespective of the way overall VaR is calculated.

Apart from developing theoretical results, we investigate the international airline industry in the 1990s to illustrate the relevance to corporate practice. This industry serves as ideal laboratory because financial risk management is inevitable given thin margins and significant external risks. We study exposures to exchange rates, jet fuel prices, interest rates and local stock market indices. The out of sample performance of the Component VaR estimates is compared to the performance of both a univariate and a multivariate estimate based on JPMorgan’s RiskMetrics™. The latter two approaches are based on the normal distribution and, since financial returns are known to be fat-tailed, they suffer from significant underestimation of risk for high VaR confidence levels. We find further evidence of this bias. Component VaR estimates, in contrast, do not suffer from such bias. The decomposition result for Component VaR yields insight in what shareholders perceive to be important sources of risk both across airlines and over time.

We focus on KLM Royal Dutch Airlines to show how Component VaR results can be interpreted. The objective of KLM’s risk management strategy is to shield shareholders from financial risks and, hence, expose them solely to business risk. In recent years this strategy was implemented and we find that the stock’s risk profile has changed accordingly. Component VaR analyses can enrich discussions on corporate risk management and shareholder value.
This paper is organised as follows. In section 2 we combine basic VaR techniques with factor models to develop the Component VaR framework. Section 3 discusses how this methodology can be applied to the airline industry. Out of sample performance is studied in section 4. To show how to interpret results, section 5 focuses on Component VaR results for KLM. Section 6 contains concluding remarks.

2 Component Value at Risk for Stocks

In this section we present the Component VaR framework. A distinguishing feature of the framework is the multi-dimensional perspective on downside risk, viz. VaR. This allows us to slice an overall VaR estimate into components. These components can be attributed to the various different sources of risk that the firm faces in the dynamic economic environment in which it operates. Using theory developed for trading portfolios to define and estimate VaR for corporate stocks is relatively straightforward. The only fundamental difference between analysing risk characteristics of trading portfolios on the one hand and individual corporate stocks on the other is the identification of the underlying risk sources and the measurement of the corresponding exposures. The variety of risk factors to which a trading portfolio is exposed is determined by the variety of financial instruments that are included in the portfolio. The portfolio exposure to these risk factors is determined by the extent to which a particular security is comprised in the portfolio. For individual stocks, however, both the identity of and subsequent exposures to risk factors are hidden. As Stulz (1996, p. 20) puts it:

“It is relatively simple to calculate VaR for a financial institution’s portfolio over a horizon of a day or a week. It is much less clear how one would compute VaR associated with, say, an airline’s ongoing operating exposure to oil prices.”

In the finance literature linear multi-factor models are used to estimate such risk exposures. Some well-known examples are Sweeney and Warga (1986) and Flannery and James (1984) who estimate the exposure of shares to the market index and the risk-free interest rate. Jorion (1990) estimates a
factor model to study exchange rate exposures of US multinationals, He and Ng (1998) do this for Japanese multinationals and Allayannis and Ofek (2001) for S&P500 non-financial firms. Tufano (1998) studies the exposures of gold mining firms to the gold price. The linearity assumption is common in the financial literature, but it would not be appropriate when measuring exposures for firms that hedge with non-linear derivatives i.e. options. This concern is mitigated by the fact that firms are inclined to use linear derivatives as is apparent from the Wharton surveys, Bodnar et al. (1998). In addition, Howton and Perfect (1998) find that 90% of interest-rate contracts are swaps. Futures and forwards comprise over 80% of currency contracts used by firms.

In general, the exposures of total stock returns to external risk sources can be estimated using the following k-factor model:

\[
\tilde{r}_t = a + \sum_{i=1}^{k} b_i \tilde{f}_{i,t} + \tilde{\epsilon}_t
\]

where:
- \(\tilde{r}_t\) = the total stock return in period t (including dividend payments),
- \(\tilde{f}_{i,t}\) = the return on underlying factor i in period t,
- \(\tilde{\epsilon}_t\) = the disturbance term.

Tildes indicate stochastic variables. The sensitivity coefficients \(b_i\) indicate the company’s exposures to the external factors as perceived by shareholders. These factors capture the relevant economic environment of the firm. The disturbance term factor \(\tilde{\epsilon}\) should be interpreted as a source of idiosyncratic or company-specific risk. The major drawback of this simple representation is that risk exposures are assumed to be constant over time. We know that exposures are subject to change for a number of reasons: (i) the company’s business activities might change and therefore its risk exposures, (ii) the company may initiate a hedging policy or change an existing policy and (iii) the perceptions of investors may change over time.
The dynamics of a company’s risk profile can be modelled by allowing for time-varying risk exposures. One could estimate the factor sensitivities by applying ordinary least squares regressions over a fixed-width window, moving in time. The major drawback of this approach is that the changing exposure might be caused by the new observation that is added to the window as well as by the one that drops out of the window. In the latter case the changing exposure is a “phantom effect” since it is not triggered by new information. In our framework we therefore choose an alternative approach: we apply exponentially weighted least squares (EWLS) regressions. If today is time $T$, the weight assigned to the observation $T-m$ is defined by

$$w_{-m} = \frac{\lambda^m}{1-\lambda}$$

where $m$ represents the number of lagged time periods and $\lambda$ is the decay factor with $0<\lambda<1$. For each value of $\lambda$ we define an effective period of $x$ time units. The length of this effective period is defined by the most recent interval $(T-x, T]$ that contains 80% percent of total weight. For example, to achieve an effective period of three years studying weekly returns we should set $\lambda$ equal to 0.99.

By applying EWLS, we track the varying risk exposures over time. We consider the regression coefficients $b_{i,t}$ estimated using information up to time $t$ as the relevant systematic risk exposures for period $t+1$ (i.e. from time $t$ to $t+1$). We therefore rewrite the factor model (1) in dynamic form:

$$\tilde{r}_{t+1} = a + \sum_{k=1}^{k} b_{k,t} \tilde{f}_{k,t+1} + b_{k+1,t} \tilde{f}_{k+1,t+1}$$

For convenience, we consider the disturbance term to be one of the factors. Without loss of generality this idiosyncratic factor $\tilde{f}_{k+1,t}$ can be scaled to obtain $b_{k+1,t} = 1$. This dynamic factor model not only offers a multi-farious view on risk but also accommodates time-varying risk exposures.
Next we consider the VaR concept with the factor model (3) in mind. Given the estimated risk exposures \( b_{ij} \) and the joint distribution of the factors (including the idiosyncratic factor) over the next period, we can construct the distribution of the stock return for the coming period. The quantile of the stock’s return distribution that meets the desired confidence limit \( c \) yields an estimate of the VaR over the next period:

\[
(4) \quad \Pr \left[ \tilde{r}_{t+1} < -r^*_t \right] = 100 - c\%
\]

where \( r^*_t \) = the stock return VaR for period t+1,

\( \tilde{r}_{t+1} \) = the stock return for period t+1,

\( c \) = the VaR confidence level.

Given this overall return VaR \( r^* \), two questions appeal to us:

1. What is the effect on the stock VaR if the exposures for one of the underlying factors were to change?

2. How can we decompose the stock VaR into components attributable to each of the underlying factors?

To answer both questions we introduce two metrics that belong to each of the risk factors: Marginal VaR and Component VaR. The Marginal VaR of factor \( i \), \( M \_VaR_i \), is defined as the change in the return VaR \( r^* \) that is caused by a marginal change in factor exposure \( b_i \):

\[
(5) \quad M \_VaR_i = \frac{\partial r^*}{\partial b_i} \quad i = 1, \ldots, k + 1
\]

The Component VaR of factor \( i \), denoted by \( C \_VaR_i \), measures the total contribution of factor \( i \) to the overall VaR. Hence we require:\(^2\)

\[
(6) \quad r^* = \sum_{i=k+1} C \_VaR_i
\]

\(^2\)Fong and Vasicek (1997) suggest another approach to decompose overall VaR into components. However, their decomposition does not satisfy equation (6).
The linearity of the factor model combined with Euler’s theorem yields a decomposition result that meets both above-mentioned criteria:

\[(7) \quad r^* = \sum_{i=k+1} b_i \cdot M \cdot \text{VaR}_i = \sum_{i=k+1} C \cdot \text{VaR}_i \]

We refer to the Appendix for details. Hence, the component of overall VaR attributable to factor \(i\) can be defined as \(b_i \cdot M - \text{VaR}_i\). We want to stress that this decomposition is a general result since it does not depend on distributional assumptions.

Now that we have established the decomposition of overall VaR we still have to estimate this overall VaR itself. This estimate is based on historical simulation, where, consistent with the estimation of factor exposures, historical share price returns are exponentially weighted with factor \(\lambda\). The empirical cumulative distribution function (CDF) that is defined by such weighting scheme has a ‘block’ structure as depicted in figure 1. The \(c*100\%\) VaR can be defined for such distribution by linear extrapolation, since every value \((1-c)*100\%\) along the vertical axis corresponds with a jump in probability from \(c\) with cumulative probability \(p\) to \(c’\) with cumulative probability \(p’\). Having estimated VaR, the decomposition result is a two step procedure. First, we determine the values of the external factors at both \(x\) and \(x’\). Based on all information up until and including the present day \(t\), we have the estimate of the exposure coefficients \(b_{i,t}\) in equation (3) using EWLS as discussed before. For both observations \(x\) and \(x’\) we can calculate \(x\) and \(x’\) as the sum of external factors multiplied by their coefficients and the error term, which is considered the idiosyncratic factor with exposure 1. In the second step we determine the decomposition result by linear extrapolation between both the decomposition for \(x\) and \(x’\). The resulting sum equals the VaR estimate. In formula terms this result can be represented as

\[(8) \quad r^*_{t+1} = a + \sum_{i=1}^{k+1} b_{i,t} (q \cdot f_i(x) + (1-q) \cdot f_i(x’)), \quad q = \frac{p’-c}{p’-p-1} \]
where \( f_i(x) \) denotes the value of factor \( i \) in observation \( x \).

The VaR estimate is efficient since it is the estimation of an order statistic. The factor estimates, on the other hand, are conditional estimates of factor values given that the return is equal to the VaR estimate. To improve the efficiency of these estimates of factor values, we base this estimate on an interval of observations around the VaR estimate. Observations are added on both sides of the VaR estimate in such a way that the weighted sum of components equals the initial VaR estimate. The appendix further details this improved estimate of the VaR decomposition. The next section discusses the empirical implementation of the Component VaR framework.

### 3 Component VaR in the Airline Industry

“An airline, for example, might find VaR helpful in assessing its exposure to jet fuel prices.”, Culp, Miller and Neves (1998)

The airline industry in this decade serves as an appropriate laboratory for the Component VaR framework. There are several reasons for this. The most important reason is that airlines are intrinsically heavily exposed to various sources of financial risk. Airline revenues are denominated in many different currencies. Furthermore, jet fuel expenses constitute a significant part of airline costs. Hence, on the operating profit level, airlines face exchange rate risk as well as commodity price risk. In addition, the degree of leverage in the industry is substantial. Considerable tax benefits can be achieved through debt financing of aircraft. Widespread use of financial lease products illustrates airlines’ interest in debt finance. This implies that at the level of net income, airlines face a substantial exposure to interest rate changes. Another reason for studying the airline industry is the creation of global airline alliances in the 1990s. Such alliances might have changed the stock profile. Finally the relatively high liquidity and volatility of airline stocks ensures sufficient share price dynamics to study.

The first step in the Component VaR framework is to identify the external risk factors and set up the factor model. We use a time horizon of one week and, hence, relate weekly share price
returns to weekly returns on exchange rates, jet fuel and government bonds. The local market index is added to account for other (omitted) factors and market sentiments that are likely to affect stock returns (cf. Burmeister and McElroy (1988)). This index is purged of the other factors’ effects by regressing the index on all other factors. The residual of this regression is added to the factor model and is referred to as the residual market index. The resulting factor model is:

\[
\tilde{r}_{t,\text{airline}} = \alpha + b_{\text{JetFuel}} \tilde{r}_{t,\text{JetFuel}} + b_{S-\text{DEM}} \tilde{r}_{t,S-\text{DEM}} + b_{S-\text{GBP}} \tilde{r}_{t,S-\text{GBP}} + b_{\text{BOND}} \tilde{r}_{t,\text{BOND}} + b_{M} \tilde{r}_{t,M} + \tilde{\epsilon}_{t}
\]

where

- \( \tilde{r}_{t,\text{airline}} \) = the total return on the airline stock in dollars,
- \( \tilde{r}_{t,\text{jet fuel}} \) = the return on jet fuel in dollars,
- \( \tilde{r}_{t,S-\text{DEM}} \) and \( \tilde{r}_{t,S-\text{GBP}} \) = the returns on the Deutschmark, a proxy for the Euro at the end of the data set, and the British Pound denominated in dollars,
- \( \tilde{r}_{t,\text{BOND}} \) = the return on the local government bond index in local currency,
- \( \tilde{r}_{t,M} \) = the residual local market index return, and
- \( \tilde{\epsilon}_{t} \) = the disturbance term.

The factor model is estimated using weekly data from the total sample period January 1st, 1990 until December 31st, 1999. We chose to study Wednesday to Wednesday returns in order to avoid potential biases from start- or end-of-the-week effects. Six major international airlines are studied: KLM Royal Dutch Airlines, British Airways, Lufthansa, American Airlines, United Airlines and Delta Airlines. These airlines were selected for two reasons: (i) these airline stocks are listed throughout the entire sample period and (ii) these stocks provide investors with a pure play on the airline business. Some other airline stocks expose investors to associated business such as the catering or travel industry.

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3 This two-step procedure where residuals generated from an auxiliary regression are used as regressors in a main regression raises some econometric issues. However, Pagan (1984) shows that the OLS estimators for the auxiliary generated residuals in the main regression are both consistent and efficient.
The data are retrieved from Bloomberg. Airline stock returns are calculated from trading prices at local exchanges. We added dividend returns to establish total shareholder return. These returns are converted into dollar returns to facilitate comparability. Jet fuel returns are derived from a Bloomberg index representing jet kerosene spot prices in dollars. Government bond returns are derived from the JPMorgan Government Bond Index, which contains all traded government bonds. For each airline we selected the local index. One drawback of taking this index bond portfolio is that the duration is likely to vary from country to country and is likely to change over time. It turns out that duration is close to 5 years for all bond portfolios and hardly changes over time. Nonetheless we adjust bond returns for duration differences by re-scaling them to a five-year duration return as follows:

\[ r_{\text{adjusted}} = 5 \cdot \frac{r_{\text{original}}}{\text{Duration}} \]

As local stock market indices for the Netherlands, Germany, the UK and US we took the AEX, DAX, FTSE100 and the NYSE Composite, respectively.

In this paper we focus on KLM Royal Dutch Airlines to illustrate what can be learned from the Component VaR results. We regress stock returns on all underlying factors for the sub-periods 1990-1992, 1993-1995, 1996-1999 as well as for the overall period. The results in table 1 show that exposures for the underlying factors are not constant over time. A LM test on equality of coefficients for these three sub-periods is rejected at a confidence level of 99%. The other airline stocks are analysed in the same way and this yields similar results. This confirms our intuition that exposures are not stationary. To track the changing exposures \( b_{t,d} \) we use the weighted least squares as described in section 2. In this approach observations are exponentially weighted with a decay factor \( \lambda \). High values of \( \lambda \) imply that the regression is based on many observations, enhancing estimation efficiency. Low values of \( \lambda \) put more emphasis on recent observations, thus enhancing
the accommodation to changing exposures. We choose $\lambda = 0.99$ to balance these two effects. This value of $\lambda$ yields an effective period of three years, since 80% of total weight falls within the last three years.

Figure 2 presents estimation results for KLM. The exposures in the first year are not very informative, since the rolling regression has too few data to estimate coefficients. Regression coefficients should be interpreted as elasticities, since a coefficient of 0.5 implies that, on average, a +10% factor return results in a +5% stock return. These results lead to the following insights:

(i) The elasticity of KLM stock for jet fuel is significant and is on average $-10\%$. In recent years the exposure has diminished. This is not surprising, since the company increased the fuel hedge in the last years of the decade to around 70% in order to gain from what it considered to be temporary low fuel prices.

(ii) As of September 1992, the elasticity of KLM stock for the dollar Deutschmark exchange rate has decreased gradually from 100% to close to 0%. The positive coefficient implies that an appreciation of the Deutschmark leads to a stronger stock value in terms of dollars. The international investor thus regards KLM as a company long in Deutschmarks. This clearly is the case when revenues are primarily generated in Europe while costs are primarily US dollar related due to fleet and fuel purchases. The coefficient diminishes over time, which is likely to be the result of KLM and the US carrier Northwest Airlines growing into an alliance across the North Atlantic. More US passengers on board of KLM flights generate increased dollar revenues and thus reduce the short position in US dollars.

(iii) The results for KLM stock exposure to the British Pound are mixed. Until the EMS crash in 1992 this exposure is significantly negative. Since then the exposure is insignificant.

(iv) The exposure to the German Government Bond has decreased over the decade from 200% positive to 100% negative. Dividend payments and fixed interest rates on debt including leases imply a positive financial exposure, since an increase in the government bond price
leads to a lower discount factor for future cash flows and therefore increases the DCF value of equity and debt. Floating rate debt neutralises this lower interest rates effect through lower interest payments. A negative exposure is related to the economic exposure of airlines to interest rates. Interest rates are correlated with the state of the economy, since interest rates are known to be high when an economy is strong and potentially overheated and low when the economy is weak. High bond prices and low interest rates therefore indicate that airlines face a strong economic environment and therefore strong traffic growth, which is positive for the share price. Over the years KLM’s exposure to the government bond has decreased which is consistent with KLM’s change from almost 100% fixed rate debt at the start of the decade to 50% fixed and 50% floating at the end of the decade.

(v) The elasticity of KLM stock to the residual local market index has decreased substantially over the decade. At the start of the decade it grew to almost 200%. Over the years it decreased to 80%. The decrease in this ‘residual beta’ is arguably due to structural changes in the industry during the decade. At the start the airline industry showed over-capacity and perfect competition. Over the years, however, the creation of world wide alliances helped it to become increasingly oligopolistic. In this case results are likely to be less sensitive to economic cycles.

4 Out of Sample Test Component VaR

Before we elaborate on the airline Marginal VaR and the decomposition of overall VaR, we study the out of sample properties of the Component VaR. It should at least do as well as a univariate and multivariate VaR estimate based on the RiskMetrics™ methodology, since this methodology is extensively tested and widely used and thus set a standard in the financial industry⁴.

⁴ Although actual RiskMetrics™ use will be difficult to estimate and as far as it is known to JPMorgan will be kept confidential, the mere fact that most financial systems (e.g. Bloomberg and Reuters) provide RiskMetrics™ or closely related functionality, proves that it has gained significant interest.
Conditional on the exposures estimates based on all information including the current week \( t \), the Component VaR is estimated one week ahead. The univariate VaR for the coming period \( t+1 \) is estimated as \( N^{-1}(c) \) times the estimate of standard deviation minus the estimated mean. \( N^{-1}(c) \) denotes the inverse of the cumulative normal density function. Both the mean and standard deviation are based on exponentially weighted historical returns up until the present time \( t \). The multivariate VaR based on RiskMetrics™ has, to our knowledge, not been defined and estimated for corporate stocks. Based on the general RiskMetrics™ theory, the multivariate VaR can be estimated in a three step procedure. First, based on all information up until and including the present time \( t \), we estimate the exposure coefficients \( b_{it} \) in equation (3) as previously discussed. Second, we estimate the means, variances and covariances, i.e. the first two moments of all factors including the residual based on exponentially weighted historical returns up until the present time \( t \). Finally, conditional on the estimated exposure coefficients and factor moments we construct a return VaR estimate for the coming period \( t+1 \). Since RiskMetrics™ assumes normality, the VaR estimate is based on the first two moments. In formula terms both the univariate and multivariate estimates are defined as:

\[
\begin{align*}
    r_{t+1}^* &= -\hat{\mu}_t + N^{-1}(c) \cdot \hat{\sigma}_t & \text{(Univariate RiskMetrics™ VaR)} \\
    r_{t+1}^* &= -\hat{\mu}_t + N^{-1}(c) \cdot \hat{b}_i \hat{\Sigma}_i \hat{b}_j & \text{(Multivariate RiskMetrics™ VaR)}
\end{align*}
\]

Furthermore, note that the univariate VaR is nested in the multivariate VaR framework since restricting the exposures to zero effectively yields a univariate estimate.

The performance of a VaR estimator is judged in terms of out of sample unbiasedness. We can think of an indicator variable \( l(t) \) which equals one if the VaR is exceeded and zero otherwise. The estimator is unbiased if the sample mean of \( l(t) \) does not differ significantly from 1 minus the VaR confidence level:

\[
\text{average}(l(t)) = 1 - c
\]
Unbiasedness is tested for four frequently used confidence levels, viz. 0.90, 0.95, 0.975 and 0.99. Empirical research summarised in the RiskMetrics™ Technical Document has shown that for daily and monthly data the optimal decay factor is 0.94 and 0.97, respectively (JPMorgan Bank (1996)). We apply the 0.94 value for the univariate VaR estimate. For the multivariate VaR we also use $\lambda=0.94$ for the covariance matrix and mean estimate. For the factor model estimates we use $\lambda=0.99$ as discussed before. Table 2 presents results for this test on the univariate and multivariate RiskMetrics™ VaR and the Component VaR. A well-known drawback of the RiskMetrics™ methodology is that it underestimates risk for high confidence levels, JPMorgan Bank (1996, p.235). We observe the same effect in our sample. For the 99% confidence level the univariate VaR significantly underestimates the VaR for three out of six airlines. The multivariate RiskMetrics™ and Component VaR, in contrast, do not lead to significant VaR underestimation. For multivariate RiskMetrics™ we find significant VaR overestimation for one airline at a 90% confidence level.

Since both RiskMetrics™ VaR estimates are based on normality, we test whether the out-of-sample-returns $r_{t+1}$ are normally distributed. To do this we standardise returns by subtracting the mean estimate and dividing by the standard deviation estimate, where both estimates are based on the information up until time $t$. The distribution of the new return series should be standard normal. The Jarque-Bera test is used to evaluate this. Figure 5 presents the results of these tests including Q-Q plots for the left tail. At a 99% confidence level the null hypothesis of normality is rejected for four out of six airlines.

To summarise, both the multivariate RiskMetrics™ and Component VaR clearly outperform the univariate RiskMetrics™ VaR on the most important criterion of unbiasedness. The Component VaR is the preferred option of these two approaches since it does not over- nor underestimate VaR at all relevant confidence levels. More importantly, it does not require the normality assumption underlying the multivariate RiskMetrics™ approach. Consistent with common knowledge, the normality assumption does not hold in our sample.
5 Component VaR Results and Interpretation

The out of sample tests show that the Component VaR performance is better than its benchmarks. The main advantage of the Component VaR as compared to a univariate VaR is that it allows for decomposition. We present M_VaR and C_VaR results for KLM Royal Dutch Airlines to show how these can be interpreted. The case of KLM is interesting since the airline has put much effort in developing a new risk management strategy in recent years. In 1998, Ernst&Young rewarded KLM in this field along with Ford, McDonalds, Microsoft and Nokia.\(^5\) A quote taken from the Ernst&Young report summarises the company's view on risk management:

“We would ultimately like to show operational risk to our shareholders and none of the financial risks.”, John de Die, SVP Finance KLM in Ernst&Young (1998)

If the company were transparent and markets perfect the risk profile of the stock should have changed accordingly. In the remainder of this section we will apply the M_VaR and C_VaR framework to study this premise.

Figure 3 illustrates how the volatility of the underlying factors developed throughout the decade. The figure shows that the fuel price is most volatile with weekly volatility peaking at 12% early 1991 which is the time of the Gulf War. It decreased to between 4 and 5% at the end of the decade. The idiosyncratic risk is second most volatile with a volatility range of 3 to 5%. Market volatility is 1 to 2 % with a sharp increase to a level of 5% by the midst of 1997 caused by unrest in international financial markets due to the Asian monetary crisis. Both the Deutschmark and British Pound exchange rate volatility and the residual market volatility are between 1% and 2%. The German Government Bond shows least volatility with levels slightly below 1%.

Figure 4 shows the results of the M_VaR analysis for KLM. The VaR confidence level is set at 95%. We left out the M_VaR for the idiosyncratic factor because the exposure to this factor is one by construction. The largest value for M_VaR occurs for jet fuel in March 1991. By that time

\(^5\) In 1998 KLM received the Ernst&Young Global Risk Manager of the Year Award.
the M_VaR for jet fuel equaled -11%. This means that being 0.1 more exposed to jet fuel results in a more than 1% increase in the stock VaR. The M_VaR for fuel decreased over time to a level closer to zero. The M_VaR for the residual market index is substantial throughout the sample. It is on average 1% with a sharp increase by the midst of 1998 ending at a level of 5% in December 1999. Increased market volatility starting at the midst of 1997 is arguably the most important reason for this rise. The marginal VaR for both the Deutschmark and British Pound exchange rate is between 0 and 2% for most of the period, but decreased to zero by the midst of 1998. The M_VaR for the German government bond is the smallest of all factors with values between 0 and 0.5%. This can be understood from the low volatility in the German Government Bond.

Figures 6a and 6b illustrate the results of Component VaR calculations for all six carriers. VaR confidence level is set at 95%. Although we study M_VaR and C_VaR for KLM, it is interesting to benchmark the stock profile development against similar airlines. We first evaluate the C_VaR results for KLM and then study the major differences as compared to other carriers.

The total weekly Value-at-Risk for KLM stock ranges from 5% to 8%. In other words, in bad times the share price lost more than 8% in one out of every 20 weeks. These turbulent times occurred at the start of the decade and overall VaR decreased over time to about a 5% loss one out of every 20 weeks by the midst of 1997. The VaR increased again to between 6 and 7% in the final year. To understand why this happened we study the development of C_VaR for each factor separately. The largest contributor to overall VaR is arguably idiosyncratic risk. This should be interpreted as risk inherent in the company’s business. In relative terms it appears to be quite stable by contributing half to overall VaR. The second largest contributor to total stock VaR is the residual market index. Its effect ranges from 1% to 3%. The dollar Deutschmark exchange rate contributes
0.25% to 1% to overall VaR. Jet fuel contribution to total risk is 2% at the start due to high volatility caused by the Gulf War. In the rest of the decade its C_VaR is negligible. The Government Bond C_VaR is substantial at the start of the decade amounting to 0.5%, which is roughly five percent of overall VaR. Its relative contribution decreased to virtually 0% by the midst of 1992. The British Pound does not contribute to overall VaR apart from the start of the decade when its contribution is substantial. This is largely due to it being linked to the Deutschmark in the EMS until 1992. Generally we find that throughout the decade commodity price, exchange rate and interest rate risk have become almost non-existent for KLM.

If we benchmark KLM Component VaR results to those of the other two European carriers, we see some remarkable differences. First, overall VaR of KLM and Lufthansa are comparable and both higher than British Airways throughout the decade. The difference, however, has become smaller over the decade. Second, apart from the first two years the relative contribution of the residual market index to total risk is largest for KLM amounting to almost half of total risk. Although its influence on VaR is also quite substantial for Lufthansa it is relatively small for British Airways, amounting to just 25% of overall VaR. British Airways might be regarded a more international stock since local market risk tends to be the smaller part of total risk.

If we now benchmark the results for European carriers to their US rivals we find that over the decade downside risk as measured by VaR developed in favour of the Europeans. Starting off with a stock VaR that was higher than that their US rivals they ended up with a lower VaR at the end of the sample period. Furthermore, we find that the C_VaR of commodity price, exchange rate and interest rate risk is relatively larger for European airlines. Differences have decreased in the course of the decade.

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6 Before September 1992 the contribution of the Deutschmark and the British Pound should be added up since both currencies being part of EMS basically means they can be regarded one source of risk.
6 Conclusion

In this paper we develop the Component Value-at-Risk framework. It is based on insights from both the VaR and multi-factor literature. We add three innovative features. First, we allow for dynamic exposures. Second, the Component VaR is non parametric and hence does not require restrictive distributional assumptions. Third, the Component VaR allows for decomposition of overall VaR into components attributable to the underlying external risk factors. This enables us to evaluate the contribution of each risk factor to the overall downside risk.

We apply the framework to analyse how shareholders perceive airline risks. Comparing this to true risk as known by the company can alert the company to potential misperceptions. We study exposures to exchange rates, jet fuel prices, interest rates and local stock market indices. Out of sample performance is compared to univariate and multivariate VaR based on the RiskMetrics™ methodology, which assumes normality. One well-known drawback of the latter approach is the significant underestimation of risk for high VaR confidence levels. We here find similar evidence for the univariate VaR estimates. Results improve when multivariate VaR estimates are tested. Both approaches, however, assume normality, which has to be rejected for most of the airline stocks. The Component VaR estimates, in contrast, do not rely on any distributional assumptions, are computationally simple and perform at least as well out of sample. Moreover, the Component VaR framework provides insight in the sources of VaR differences across airlines and in VaR changes over time.

We focus on KLM Royal Dutch Airlines to show how Component VaR results can be interpreted. The objective of KLM’s risk management strategy is to shield shareholders from financial risks and, hence, expose them solely to business risk. In recent years this strategy was implemented and we find that the stock’s risk profile has changed accordingly.
References


Ernst&Young, 1998, “The 1998 Ernst&Young Global Risk Manager of the Year Award”, Ernst&Young US.


Appendix: General linear decomposition of overall VaR

In this Appendix, we derive a decomposition of overall VaR into components. This decomposition result is of a very general nature since it does not depend on any distributional assumptions that may be needed to estimate the overall VaR. The results derived below prevail as long as the security return can be expressed as a linear combination of the factor returns. In addition we show how Marginal and Component VaRs may be estimated in a non-parametric setting. For notational simplicity we suppress the time index \( t \) in the following.

Deriving Marginal VaR and Component VaR

We assume that all relevant return distributions have finite first moments. According to the linear k-factor model equation (3), the security return \( \tilde{r} \) is a linearly homogeneous function of the intercept \( \alpha \) and the factor sensitivities \( \{b_i\}_{i=k+1} \). Since this function is continuous and analytic we have by the very definition of conditional expectations:

\[
\tilde{r} = E\left\{ \tilde{r} \mid \tilde{r} = \alpha E \left\{ \frac{\partial \tilde{r}}{\partial \alpha} \right\} + \sum_{i=k+1} b_i E \left\{ \frac{\partial \tilde{r}}{\partial b_i} \right\} \right\} = \alpha + \sum_{i=k+1} b_i E \left\{ f_i \mid \tilde{r} \right\}
\]

where the first equality follows from applying Euler’s theorem. When the security return takes the particular value \( -r^* \), the conditional expectation becomes deterministic:

\[
r^* = -\alpha - \sum_{i=k+1} b_i E \left\{ \frac{\partial \tilde{r}}{\partial b_i} \mid \tilde{r} = -r^* \right\} = -\alpha - \sum_{i=k+1} b_i E \left\{ f_i \mid -r^* \right\}
\]

Note that the conditional expectation in the first equality indicates the Marginal VaR of factor \( i \):

\[
(A-3) \quad M_{VaR_i} = -E \left\{ f_i \mid -r^* \right\} \quad i = 1, ..., k + 1
\]

Furthermore, since the component VaR of factor \( i \) measures the total contribution of factor \( i \) to the overall VaR, we have:

\[
(A-4) \quad C_{VaR_i} = -b_i \cdot E \left\{ f_i \mid -r^* \right\} \quad i = 1, ..., k + 1
\]
In an efficient market only unexpected factor changes are relevant. Hence the unconditional expectations of all factor shocks are zero:

\[ E(\tilde{f}_i) = 0 \quad i = 1,\ldots,k+1 \]

This implies that the intercept of the factor model represents the mean stock return: \( \alpha = E(\tilde{r}) \). Thus the marginal VaR of the intercept is minus one, indicating that an increase of the mean stock return with x% will lower overall VaR with x%. Likewise, the component VaR of the intercept is simply minus the mean stock return. Since the mean return is oftentimes very small compared to the volatility of the factors, the component VaR of the mean return is dwarfed by the factor component VaRs.

**Estimating Marginal VaR and Component VaR**

We use exponentially weighted historical simulation to estimate the security’s overall VaR. Given today’s time \( t \), the weight assigned to observation \( t - m \) is \( \lambda^m (1 - \lambda)^{-1} \). For estimating the overall VaR with confidence limit \( c \) we determine the lower the \( 1 - c \) quantile of the security’s historical return frequency distribution. We start from the data matrix whose rows contain the security return and the corresponding realized factor returns. Taking the security return as the sort key, we next order the rows of the data matrix from the lowest security return to the highest. Starting from the lowest security return we move up while adding the weights assigned to the corresponding returns. When the sum of the weights exceeds \( 1 - c \) we linearly interpolate between that particular return and the return immediately below it (see figure 1). Minus the interpolated return is the overall VaR \( r^* \).

Eq.(A-3) indicates that the security’s marginal VaR with respect to factor \( i \) can be estimated by minus the factor’s mean return conditional on the security return being equal to \( r = -r^* \). Having determined the two rows from which the overall VaR is interpolated, we could apply the same interpolation recipe to the values of factor \( i \) in the corresponding data rows. Minus
the interpolated factor return would then indicate the marginal VaR with respect to this factor. Since we are estimating the conditional expectation from only two observations, however, this procedure is prone to estimation risk. In order to mitigate the effect of this estimation risk, we estimate the conditional means in equation (3) by using $\tau$ observations in a window of the ordered data matrix selected around $r = -r^*$. For the VaR confidence levels of 90% and 95% we use $\tau = 10$. For 97.5% and 99% we use $\tau = 5$ and $\tau = 3$, respectively.

For determining the appropriate location of the window of $\tau$ ordered observations we employ the following procedure. Having selected some data window of size $\tau$ around $r = -r^*$ we compute the weighted average of the security returns within that window. This is the overall VaR implied by the $\tau$-window. Next we compare this “window VaR” with the overall VaR which was estimated from the two interpolated weighted security returns. When the window VaR is higher (lower) than the overall VaR we shift the data window to the right (left), i.e. in the direction of the higher (lower) security returns. We then calculate the new window VaR. We repeat this procedure until the weighted average of the security returns within the $\tau$-window is equal to minus the estimated overall VaR. On the basis of the appropriate window identified in the last step we estimate Marginal VaRs and Component VaRs. The Marginal VaR of factor $i$ is estimated by the weighted average of its returns within the window. For estimating the Component VaR of factor $i$ we multiply its Marginal VaR with the estimated factor sensitivity according to equation (A-4).

Validity of the results

We have derived estimators for Marginal VaR and Component VaR assuming only linearity of the underlying return generating process. The decomposition of overall VaR is a general result and hence does not depend on any distributional assumption. Our analysis extends the parametric (i.e. multivariate normal) case as considered by Garman (1996) and Jorion (1997, p.154).
**Table 1: Determinants of KLM Share Price**

*Significance levels: *: Significant at the 90% level, **: Significant at the 95% level*

<table>
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<th></th>
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</thead>
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<tr>
<td><strong>Constant</strong></td>
<td>-0.08</td>
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<td>(-0.23)</td>
<td>(0.96)</td>
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<td>(0.19)</td>
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<td>(8.86)</td>
<td>(7.39)</td>
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<td>(12.50)</td>
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**σ (weekly KLM return, $)**: 4.87% 4.21% 4.65% 4.60%

| **Number of Observations** | 155 | 156 | 209 | 520 |
| **R^2**                    | 0.45 | 0.36 | 0.20 | 0.28 |

This table shows the results of a regression with the weekly KLM total share return in dollar as the dependent variable. Explanatory factors are: jet fuel return, the $-Deutschmark exchange rate, the $-British Pound exchange rate, the return on the German government bond index and the residual local market index.

The value of the LM test statistic that tests for equality of coefficients for the three subperiods is 25.5, which is higher than the appropriate $\chi^2(10)$ critical values (23.21 for 99%, 18.31 for 95% and 15.99 for the 90%). This indicates that exposures of KLM to the underlying factors is not constant overtime.
Table 2: Out of Sample test VaR

\[ \lambda(\text{regression}) = 0.99, \lambda(\text{mean}) = 0.94, \lambda(\text{variance}) = 0.94, \lambda(\text{hist. sim.}) = 0.99 \]

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<th>Multivariate RiskMetrics™</th>
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<td>Number of times VaR is exceeded</td>
<td>t-value</td>
<td>Number of times VaR is exceeded</td>
<td>t-value</td>
<td>Number of times VaR is exceeded</td>
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<td>-</td>
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<td>0.0085</td>
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</table>

**+**: Significant overestimation of risk at 95% confidence level

**-**: Significant underestimation of risk at 95% confidence level

This table shows the results for an out of sample test on the univariate RiskMetrics™ VaR, the multivariate RiskMetrics™ VaR and the non parametric Component VaR. The test statistic is the number of times VaR is exceeded divided by the total number of observations. Exceeding VaR is defined as the return in the interval \( (t, t+1) \) being smaller than the out of sample VaR estimate. The non parametric VaR estimate is based on historical simulation where historical observations \( s (s \leq t) \) is assigned a weight equal to \( \lambda^s \). For both the univariate and multivariate RiskMetrics™ approach the VaR estimate is defined as \(-N^{-1}(c)\sigma_t\), where \( \sigma_t \) is the conditional standard deviation estimate based on information up to and including time \( t \), \( c \) is the VaR confidence level. The univariate and multivariate RiskMetrics™ VaR approach differ in the way this \( \sigma_t \) is estimated.

The null hypothesis is that the test statistic is equal to one minus the confidence level of the VaR measure.
This figure illustrates how Component VaR is estimated based on historical simulation. The empirical cumulative distribution function (CDF) is based on exponentially weighted historical returns.
These figures show the exposure of KLM stock to external factors. Exposures are estimated for the period January 1, 1990 until December 31, 1999. A rolling weighted least squares method is used, whereby recent observations are considered more valuable than remote observations. An exponential scale is applied with $\lambda$ equal to 0.99.
Figure 3: Volatility Underlying Factors

\[ \lambda \text{(variance)} = 0.94 \]

This figure shows return volatilities for the underlying factors. The covariance matrix is calculated using historical returns and applying exponential weights. The \( \lambda \) is equal to 0.94.

Figure 4: Marginal Value at Risk

KLM Royal Dutch Airlines

\[ \lambda \text{(regression)} = 0.99, \text{ VaR Confidence Level} = 95\% \]

This figure shows the Marginal Value at Risk for the KLM share for the period January 1, 1990 until December 31, 1999. The marginal VaR is calculated in a two step approach. First, the sensitivities of the KLM share for the explanatory factors are estimated. A rolling weighted least squares method is used, where recent observations are considered more valuable. An exponential scale is applied with \( \lambda \) equal to 0.99. Second, the Marginal VaR is calculated based on techniques discussed in the appendix. This \( M \_\text{VaR} \) is the marginal contribution of each factor to overall VaR. The VaR confidence level is set at 95\%.
Figure 5: Out of Sample Test on Normality: 
Univariate and Multivariate RiskMetrics™ VaR

\[ \lambda(\text{mean}) = 0.94, \ \lambda(\text{variance}) = 0.94, \ \lambda(\text{regression}) = 0.99 \]

This figure illustrates the out of sample distribution of the airline shares. Estimates for mean and volatility are based on the observations until and including time \( t \). The airline stock total return from \( t \) to \( t+1 \) is standardised according to these estimates. The resulting series is assumed to have a normal, standardised distribution. A Q-Q plot for the left tails, the distribution's mean, standard error, skewness and kurtosis and a Jarque-Bera test on normality can be found in this figure. The left-hand side shows the results of a univariate RiskMetrics™ approach, returns standardised according to an exponentially weighted average mean and a RiskMetrics™ volatility estimate (\( \lambda = 0.94 \)). The right-hand side shows the results of a multivariate RiskMetrics™ approach, returns standardized according to a conditional mean and volatility based on a factor model estimate and a RiskMetrics™ variance matrix estimate (\( \lambda = 0.94 \)).
This figure shows the Component VaRs for European airlines for the period January 1, 1991 until December 31, 1999. The Component VaR is calculated in a two step approach. First, the sensitivities of the stock for the explanatory factors are estimated. A rolling weighted least squares method is used, whereby recent observations are considered more valuable. An exponential scale is applied with $\lambda$ equal to 0.99. Second, the Component VaR technique is applied to find the individual contribution of each factor to overall VaR. The VaR confidence level is set at 95%. 
This figure shows the Component VaRs for US airlines for the period January 1, 1990 until December 31, 1999. The Component VaR is calculated in a two step approach. First, the sensitivities of the stock for the explanatory factors are estimated. A rolling weighted least squares method is used, whereby recent observations are considered more valuable. An exponential scale is applied with $\lambda$ equal to 0.99. Second, the Component VaR technique is applied to find the individual contribution of each factor to overall VaR. The VaR confidence level is set at 95%.