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Credit Risk Modeling and Internal Capital Allocation Processes: Implications for a Models-Based Regulatory Bank Capital Standard

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This paper surveys the current state-of-the-art in credit risk modeling at large U.S. banks. Within this context, the paper examines the near-term feasibility of an internal models approach to setting formal regulatory capital requirements for banks, as a replacement for the 1988 Basle Accord. Such an overhaul of the international capital standards would require, in our view, specific attention to several deficiencies in current modeling practices, including questions relating to model specification, parameter estimation, and model validation procedures. The paper also discusses possible uses of internal risk models for setting regulatory capital requirements against *selected* credit instruments and/or improving examination guidance dealing with the capital adequacy of large, complex banking organizations. © 1999 Elsevier Science Inc.

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I. Introduction and Summary

Nearly a decade has passed since the 1988 Basle Capital Accord established the basic architecture for setting minimum risk-based capital (RBC) requirements for banking organizations (banks). The Accord's initial objectives, achieved relatively quickly, were both to provide cross-border consistency in capital standards and to increase the capital cushions of the world's largest banks. Along with this early success has come heightened

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reliance on capital-based regulatory and supervisory policies. Within the United States, for example, Prompt Corrective Action and other provisions of the FDIC Improvement Act of 1991 now link supervisory and regulatory policies explicitly to banks' regulatory capital ratios. Thus, RBC ratios have come to be viewed as important indicators of bank soundness and, by extension, important devices by which to reach prudential regulatory objectives, such as limiting the real resource cost of bank insolvency [for a discussion on the objectives of capital regulation, see Berger et al. (1995)]. But, even as the formal RBC ratios have assumed great prominence, ongoing technological and financial innovations have exposed shortcomings in the Basle framework that, if not redressed, could undermine the future role of bank capital standards.

The most significant flaws in the RBC standards have long been recognized. First, the measures of capital embodied in the numerators of these ratios may not represent accurately a bank's capacity to absorb either expected or unexpected losses. Loan loss reserves, for example, often tend to exceed expected credit losses during good times and to understate expected credit losses during times of stress. Second, the denominator of these ratios, total risk-weighted assets, is not an accurate measure of total risk. In addition to ignoring certain risks, such as interest rate and operating risk, the regulatory risk-weights ignore differences in credit risk among financial instruments (e.g., all commercial credits incur the same 100% risk-weight or, equivalently, an 8% total RBC requirement). The risk-weights also ignore differences across banks in portfolio diversification, hedging activities, and the quality of internal risk management systems.

Spurred by opportunities for regulatory capital arbitrage created by the above anomalies, securitization and other financial innovations are rendering the formal RBC ratios increasingly less meaningful, at least for the largest, most sophisticated banks.¹ Through securitization, in particular, large banks have lowered their RBC requirements substantially without reducing materially their overall credit risk exposures. More recently, the September 1997 Market Risk Amendment to the Basle Accord has created additional capital arbitrage opportunities by permitting banks to use their Value-at-Risk (VAR) models for calculating RBC requirements against specific risks within their trading portfolios. Under this amendment, a bank could potentially reduce its RBC requirement against certain credits from 8% to much smaller amounts by shifting these exposures from its banking book to its trading account.

With the formal RBC ratios rendered less useful, judgmental assessments of capital adequacy through the examination process necessarily have assumed heightened importance. Yet, this process, too, has become more problematic as regulatory capital arbitrage has made credit risk positions less transparent. Although examination assessments of capital adequacy normally attempt to adjust reported capital ratios for shortfalls in loan loss reserves relative to *expected* future charge-offs, examiners' tools are limited in their ability to deal effectively with *credit risk*—measured as the *uncertainty* of future credit losses around their expected levels.

The academic literature on credit risk has tended to concentrate on estimates of default probability for groups of banking assets [see Altman and Saunders (1997) for a review of

¹ The basic arbitrage techniques involve: 1) re-engineering financial contracts to convert a bank's on-balance sheet credit risk into a nearly equivalent off-balance sheet exposure having a lower capital requirement; and/or 2) removing from the banking book financial instruments for which the 8% Basle capital standard is too high, relative to the underlying economic risks, while retaining instruments for which the Basle standard is too low (termed "cherry-picking"). See Jones (1998).

this literature]. That is, researchers have concentrated on credit scoring and similar models, in which the determinants of default (or loss) over a given time horizon have been estimated. But estimates of the shape of credit loss *distributions* over particular horizons have been primarily the province of market practitioners—see, most notably, Gupton et al. (1997) and Credit Suisse (1998). This work represents a natural response to increased domestic and international competition and the greater complexity of banks' credit portfolios, which now include various forms of credit derivatives.

Analogous to trading account VAR models, internal credit risk models are used in estimating the *economic* capital needed to support a bank's credit activities. By design, these systems create strong incentives for managers to economize on costly equity capital. Internal capital allocations are the basis for estimating the *risk-adjusted* returns (on allocated capital) of various bank activities which, in turn, are used in evaluations of managerial performance and in determinations of managerial compensation. Economic capital allocations also have been incorporated into risk management processes, including risk-based pricing models for credit products, the setting of portfolio concentration and exposure limits, and day-to-day portfolio management.

In principle, the inputs or outputs of a bank's internal systems for measuring risk and allocating capital could provide valuable information for use in prudential assessments of bank capital adequacy. Potentially, such assessments could be made more incentive-compatible and more risk-focused. Enhancements to banks' risk management systems might translate more quickly into improved prudential policies and procedures, perhaps reducing incentives to "game" supervisors by channeling credit risk through opaque, off-balance sheet transactions. The use of internal risk estimates by supervisors also might promote more rapid development of improved risk management techniques, and faster convergence toward a common risk measurement framework and language, which could lead to improved risk disclosures (greater transparency). Such hypothetical benefits, of course, are predicated on the reliability of internal risk models.

This paper evaluates the near-term feasibility of an internal models approach to assessing RBC for the *banking book*.² In this context, we believe it is important to distinguish between: 1) formal minimum *regulatory* capital standards, as embodied in the Basle Accord, and 2) discretionary *supervisory* assessments of capital adequacy. Within the United States, in principle, a bank's effective minimum RBC requirement is determined on a case-by-case basis through the examination process, taking into consideration the bank's asset quality, portfolio diversification, risk management practices, and other relevant factors. To promote greater consistency within this process, the regulatory capital standards are viewed as establishing a capital floor for a hypothetical bank having *excellent* asset quality, diversification, and management practices. That is, the regulatory minimums establish a common benchmark or frame-of-reference to which examiners may then apply appropriate add-ons when assessing the capital adequacy of a particular bank.

Although banks' internal credit risk models could, in theory, be incorporated into this framework through either changes in regulatory RBC standards (i.e., the Basle Accord) or

² The distinction between the banking book and the trading account is essentially an accounting distinction. Large banks' credit risk models, in principle, might be used wherever credit risk is incurred. In regulatory practice, however, some large banks are now permitted to use models to set regulatory capital for market and certain credit risks within the trading account. Therefore, the focus of this paper is on the potential use of models and, in particular, credit risk models within the banking book, which traditionally includes certain activities (e.g., commercial and consumer lending) not generally considered part of the trading account.

changes in the procedures used by examiners for determining discretionary add-ons, in practice, these alternatives tend *not* to be viewed as equivalent. Because formal regulatory capital ratios are the basis for public disclosures and are tied explicitly to certain supervisory and regulatory policies (e.g., prompt corrective action), they tend to attract closer public scrutiny. In addition, by establishing an absolute floor for what is considered an acceptable level of capital, the regulatory capital standards may, in some circumstances, unduly limit supervisory actions. Partly to preserve supervisory flexibility, the U.S. banking agencies generally have refrained from providing highly-specific written examination guidance for assessing capital adequacy. One drawback, however, is that examiners may have a limited objective basis, and consequently limited scope, for compelling a bank to maintain capital ratios significantly above the regulatory minimums, or those of its competitors, unless the institution is clearly troubled.³ In this light, changes in the regulatory capital standards may be more effective than changes in discretionary examination practices for promoting market discipline and sound risk management practices.

In dealing with these tradeoffs, the 1997 Market Risk Amendment established several qualitative standards that must be satisfied in order for a bank's trading account VAR model to be used in calculating its regulatory RBC requirement against market and specific risks. These qualitative standards require, in effect, that a bank be able to show that its risk measurement process is reliable and implemented with integrity. Among the factors considered by examiners in judging compliance with these standards are whether the bank's risk model is analytically sound, subject to periodic backtesting and stress testing, and well-integrated into the bank's management decision making process [see Hendricks and Hirtle (1997)]. Below, we use these criteria as a template for examining the reliability and integrity of the current generation of credit risk models at large banks.

To anticipate this discussion, in our view, rapid movement to an internal models approach for setting regulatory RBC requirements against the *banking book*—as a replacement for the Basle Accord—would be premature given the *current* state-of-the-art. Especially problematic is the subjectivity inherent in current credit risk modeling practices which, in conjunction with limitations of current back-testing and other validation techniques, should raise important concerns regarding the ability of banks (or supervisors) to assess model reliability in an objective manner. Although qualitatively similar concerns were raised in the context of VAR models for the *trading account*, the magnitude of these problems may be much greater with respect to risk models for the *banking book*.

An equally important issue is the treatment of operating risk within these systems. Although operating risk is not considered explicitly within the Basle framework, at many large banks the amount of economic capital allocated against operating risk is comparable to that allocated against credit risk. Models for estimating operating risk, though, are relatively primitive compared with those for credit and market risks. Before adopting an internal models alternative to the Basle Accord, supervisors will need to consider carefully whether or how to incorporate operating risk within the formal RBC framework.

³ Even in the absence of regulatory capital arbitrage, banks might wish to maintain capital above the Basle minimums, if for no other reason than to absorb short-term losses without having to incur a regulatory call for issuance of (dilutive) new capital. In practice, formal supervisory actions calling for additional capital generally occur: 1) after a bank has incurred significant losses which drive capital below certain trigger levels, or 2) after, as the result of examination, a bank is required to write down assets and/or add to loan-loss-reserves against troubled assets, thus driving reported capital below the trigger levels.

Given the rapid progress which is occurring in risk modeling techniques, it is conceivable that further improvements in this area could redress many, if not most, of the model reliability concerns raised in this paper. Although an internal models replacement for the Basle Accord may take some time, we nevertheless believe that supervisors have little choice but to develop more effective methods for assessing capital adequacy if capital-based prudential policies are to remain viable. The scale of regulatory capital arbitrage undertaken by the largest banks is indicative of the distortions created by the Basle framework, and portends continued erosion of the current RBC ratio measures as useful indicators of bank safety and soundness.⁴ As the most accurate information regarding a bank's risk is likely to reside within its own internal risk measurement and management systems, it seems clear that supervisors should utilize this information to the extent possible.

To this end, we suggest possible near-term roles for incorporating internal risk models into prudential capital policies. Specifically, internal credit risk models may be useful in: 1) the setting of *regulatory* RBC requirements against *selected* instruments for which the Basle Accord is largely silent or ineffectual, such as recourse and certain other credit enhancements supporting bank securitization activities; and 2) the development of specific and practical *examination* guidance for assessing the capital adequacy of large, complex banks.⁵

The remainder of this paper is organized as follows. To provide a context for later discussions of internal credit risk models, Section II presents a general overview of the economic capital allocation systems at large U.S. banks, while Sections III and IV summarize the range of credit risk modeling practices. Section V raises several criticisms of the internal models which may be of concern to supervisors. Lastly, in Section VI, we discuss possible near-term regulatory and supervisory uses of internal risk models.

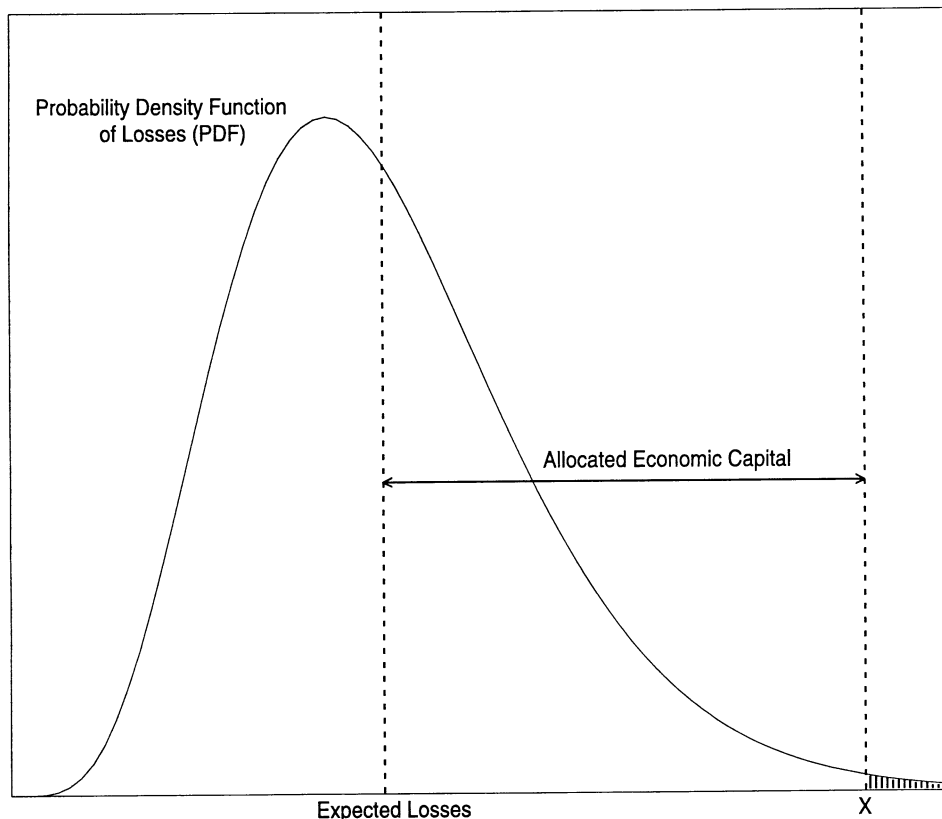
II. Overview of Economic Capital Allocation Systems

As used in this paper, the term "risk model" refers to all of the procedures employed by a bank to quantify its economic risks, whether with respect to a single transaction or to a group of transactions, customers, or products. Such estimates are used internally to allocate economic capital (defined below) to activities based on their estimated contributions to the bank's total risk—of which credit risk is one component. This section describes, in general terms, the internal economic capital allocation systems used by major U.S. banks. While such systems typically encompass all forms of risk facing a bank (credit, market, and operating risks), our principal focus is on the allocation of economic capital for credit risk.

One can think of the large banks' economic capital allocation systems as embodying either an explicit or implicit estimate of the *probability density function of credit losses* (PDF) for a bank's credit portfolio or sub-portfolio. Exhibit 1 illustrates such a PDF. As

⁴ For example, we estimate that, as of December 31, 1997, the outstanding securitized assets of large U.S. banks exceeded \$800 billion. As discussed in Jones (1998), a substantial portion of the credit risk of these assets was retained by the sponsoring banks, although the Basle capital standards do not generally fully recognize this risk retention.

⁵ And, of course, no matter how improved their treatment of *credit risk*, minimum capital standards will be ineffective unless loan loss reserves accurately reflect *expected* credit losses. See Jones and King (1995).



Losses

Exhibit 1. Relationship between PDF and allocated economic capital. *Note:* The shaded area under the PDF to the right of X (i.e., the target insolvency rate) equals the probability that unexpected losses will exceed the allocated economic capital.

discussed in Section IV, the precise definitions of credit loss tend to vary across banks, depending on their conceptual frameworks for valuing credit instruments for risk measurement purposes (e.g., a mark-to-market paradigm). For a given definition of credit losses, a risky portfolio, loosely speaking, is one with a PDF which has a relatively long, fat tail—that is, where there is a relatively high likelihood that losses will be substantially higher than mean, or expected, losses. Although in this section we treat the PDF as given, later sections of the paper will examine the techniques used in its estimation and validation.

For purposes of internal decision making, banks generally collapse the estimated PDF into a single metric, termed the “economic capital” allocation for credit risk. This process is analogous to the VAR methods used in allocating economic capital against market risks. Specifically, the economic capital allocation is determined (in theory) so that the probability of unexpected credit losses exhausting economic capital is less than some targeted level. For instance, the level of economic capital may be set to achieve a 0.03% estimated

probability that unexpected credit losses will exceed this level, thereby causing insolvency.⁶ The target insolvency rate usually is chosen to be consistent with the bank's desired credit rating for its liabilities: if the desired credit rating is AA, the target insolvency rate might be set at the historical default rate for AA-rated corporate bonds (about 3 basis points).

Within economic capital allocation systems, a critical distinction is made between *expected* credit losses and the *uncertainty* of credit losses or *credit risk*. These systems generally assume that it is the role of reserving policies to cover expected credit losses, while it is the role of equity capital to cover credit risk. In Exhibit 1, therefore, the area under the PDF to the left of expected losses should be covered by the loan loss reserve, and the bank's required economic capital is the amount of equity over and above expected losses necessary to achieve the target insolvency rate. Under this framework, a bank would consider itself to be undercapitalized if its tangible equity was less than its required economic capital. Implicit in this calculus is the presumption that the bank's reserving policies produce a net carrying value for the loan portfolio that approximates its underlying economic value.⁷

Economic capital allocations tend to be used in two broad areas: 1) the measurement of risk-adjusted profits, and 2) the management of portfolio risks. An activity's risk-adjusted profits typically are measured by adjusting traditional cost-accounting measures of net income for the opportunity cost of the equity needed to support that activity, where the amount of supporting equity is the economic capital allocated to that activity. Specifically, risk-adjusted profits would be calculated as revenues allocated to the activity, *less* the cost of allocated debt, *less* allocated non-interest expense (including an expense for expected credit losses), *less* the cost of allocated economic capital. The cost of economic capital is defined as the activity's allocated economic capital *times* the bank's ROE target or hurdle rate.⁸

In this fashion, risk-adjusted profits for various activities (or what many banks call shareholder value-added) can be placed on an apples-to-apples basis. Armed with this information, managers can make informed decisions about how to allocate scarce resources—that is, they can determine which activities to increase in size or scope, which to cut back, which to eliminate. Increasingly, measures of risk-adjusted profitability also are used in determining managerial compensation.

The second broad application for economic capital allocations is in the area of risk management, both at the level of individual transactions and at the level of the overall portfolio. When setting the price on a proposed new loan facility, it is now fairly common for a banker to first determine the break-even interest rate needed to cover the loan's *expected losses* and an appropriate margin for *credit risk* or *unexpected losses*—deter-

⁶ The economic capital allocation for a sub-portfolio of credit instruments would be calculated as the difference between the portfolio's economic capital allocations with and without the inclusion of those particular assets.

⁷ To risk modelers, the loss reserve's primary role is to provide the necessary adjustment between the carrying value of loans and their underlying economic value implicit in the credit risk model. To the extent the actual loss reserve exceeds (falls short of) this amount—perhaps because the loss reserve is actually set to accommodate regulatory or GAAP notions of acceptable reserves—the excess reserve (reserve shortfall) is treated as equity (negative equity) capital for purposes of assessing the adequacy of equity in relation to economic capital.

⁸ For purposes of calculating risk-adjusted profits, economic capital reflects all forms of risk—credit, market, and operating risk. In practice, the total economic capital for an activity is generally calculated as the simple summation of the separate allocations for each type of risk.

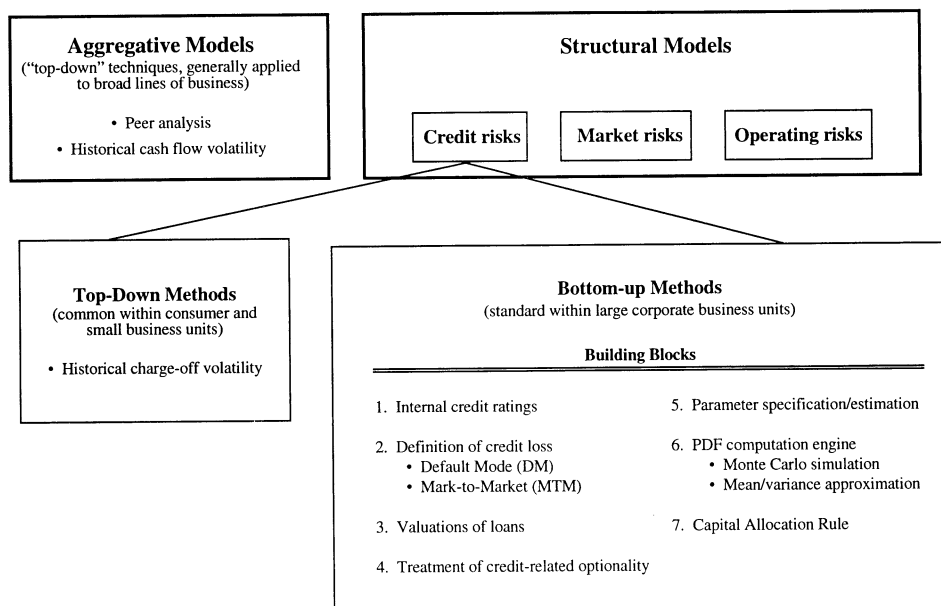


Exhibit 2. Overview of risk measurement systems.

mined so that the expected rate of return on the capital allocated to the loan (the risk-adjusted return on capital, or RAROC) achieves the bank’s hurdle rate. If other market participants are charging a lower interest rate on such loans than is necessary to meet the bank’s RAROC hurdle rate, the banker may decline and send the loan business elsewhere. Or, the banker may treat the loan as a loss-leader and hope to make up the difference via other, non-loan business with that customer.

Quite apart from determining appropriate risk-based pricing on individual loans, banks also use credit risk models in active portfolio management. To give one example, some banks use credit risk models to estimate an efficient portfolio frontier, defined as all feasible combinations of the mean and variance of portfolio rate-of-return showing, for a given mean, the lowest achievable variance. By comparing this frontier with the mean and variance of the actual portfolio, risk managers are able to develop strategies for altering the current portfolio to achieve a more preferred risk-return profile. This might be accomplished, for example, by modifying the pattern of new loan originations, by buying/selling loans in secondary markets, or by undertaking credit derivative transactions to lay off (or acquire) various credit exposures.

III. Approaches to Risk Measurement: Aggregative vs. Structural Models

Among the largest U.S. banks, there is great diversity in risk modeling practices. To provide a taxonomy for discussion purposes, we have divided risk measurement approaches into two broad categories: aggregative models and structural models, illustrated in Exhibit 2. This section presents an overview of these alternative methodologies.

Aggregative Risk Models

Aggregative models typically are top-down approaches which attempt to infer the *total risk* (i.e., the sum of credit, market, and operating risks) of a broadly-defined business or product line from the capital ratios of peers or from the historical volatility of the cash flows associated with that activity. Peer group or market comparables analysis attempts to estimate the capital which would be needed to achieve a hypothetical target credit rating for a given activity (as if operated on a stand-alone basis) from the capitalization rates of competitors engaged in that activity. Typically, this approach is applied only to complete lines of business or broad product grouping (e.g., credit cards), for which data on publicly-traded competitors are readily available. The analysis is market-based in assuming that the observed capital levels of peers reflect effective market discipline and are, therefore, about right. To better ensure apples-to-apples comparisons, capital ratios within the peer group usually are adjusted to remove the estimated effects of: 1) various accounting distortions, such as securitization; 2) disparities between the bank's desired credit rating (e.g., AA) and the actual credit ratings of peers; and 3) broad differences in portfolio composition (e.g., variations in the relative sizes of consumer versus C&I lending).

The other major aggregative technique, historical cash flow analysis, attempts to estimate an activity's total risk from the volatility of its historical cash flows. Implicitly, historical cash flow volatility (per dollar of notional size) is assumed to equal future volatility. To minimize implementation costs to the bank, the underlying cash flow estimates generally are constructed from raw data already generated within the bank's management information systems (MIS), again, usually for broad product groupings. Adjustments normally are applied to these raw data so that the cash flow for a period (e.g., a quarter or a year) can be interpreted as an approximation to the activity's economic earnings, sometimes termed Net Operating Profit After Tax or "NOPAT." Given a time series of historical NOPAT, the total risk of an activity (per dollar of notional size) is estimated by the standard deviation of the historical ratios of NOPAT to notional size.

While aggregative models for allocating economic capital are quite common among nonfinancial firms for which operating risks predominate, they are less prevalent among banks, which are affected more significantly by credit and market risks. Among banks, aggregative models tend to be used mainly for assessing the performance of broad business or product lines, for making large-scale strategic business decisions (such as acquisitions or divestitures), or for validating structural risk models, rather than for day-to-day investment and risk management purposes.

This pattern of usage reflects two perceived limitations of aggregative models. First, as noted above, data availability often makes it difficult to apply these models at the level of individual transactions or customer relationships (e.g., in product pricing decisions). A second drawback is their relative insensitivity to variations in portfolio composition within the business lines that are separately analyzed. Peer analysis, for example, may be misleading if the credit quality of a bank's portfolio differs significantly from that of its competitors. Similarly, the historical cash flow approach may be inappropriate if the composition of the current portfolio (e.g., its sectoral make-up or the credit quality of the underlying customers) is substantially different from that historically.

Structural Risk Models

In contrast to aggregative models, structural modeling approaches estimate total risk through the separate modeling of credit, market, and operating risks.⁹ With respect to the modeling of credit risk, most banks use multiple approaches within the organization. Quite often, a bank may employ top-down approaches in certain lines of business (e.g., consumer or small business lending), and bottom-up approaches in others (e.g., large corporate customers).

For consumer and small business customers, the estimation of credit risk is often carried out using top-down methods similar to those described above. That is, for purposes of internal capital allocations, a bank would assume that certain broad classes of loans (e.g., credit cards) are more or less homogeneous, and that the associated PDF can be estimated from the bank's historical net charge-off experience on such credits. In some cases, to obtain more accurate estimates, a bank may pool its own historical credit loss experience with those of peers (derived from public financial statements and Call Reports). Top-down credit risk models generally are vulnerable to the same concerns as top-down aggregative models, relating to changes over time in portfolio composition.

Within banks' large corporate businesses, credit risk normally is modeled using bottom-up approaches. That is, the bank attempts to identify and model risk at the level of each individual credit facility (e.g., a loan or a line of credit) based on explicit evaluations of the financial condition of that customer. To measure risk at higher levels of consolidation, such as for a customer relationship or a line of business, these individual risk estimates are summed taking into account diversification effects. Thus, within bottom-up models, variations in credit quality across customers and other compositional effects are considered explicitly.

IV. Credit Risk Models: Building Blocks

This section presents a description of the main building blocks of (bottom-up) credit risk models used within banks' large corporate business units, including the following elements of the credit risk modeling/capital allocation process: 1) the internal credit rating system; 2) the definition of credit losses; 3) the valuation of loans; 4) the treatment of credit-related optionality; 5) the specification/estimation of model parameters; 6) the calculation of the PDF, and 7) the choice of economic capital allocation rules.

Internal Credit Rating System

Bankers have long recognized that knowing your customer is the first line of defense against credit losses. To this end, all but of a few of the top-50 U.S. banks assign a credit rating to each large- and middle-market customer, as well as to each customer's separate credit facilities—defined to include all on- and off-balance sheet credit exposures.¹⁰ Internal credit rating systems are designed to differentiate the credit quality of borrowers much more finely than under the five-point grading scale used by bank examiners (i.e.,

⁹ Although the word "structural" is used by some practitioners as in the context above, a referee points out that such models, in fact, are disaggregative, not structural, in that they tend to ignore the structural relationships among the types of risks.

¹⁰ Some banks also assign credit ratings to consumer and small business customers.

pass, specially mentioned, sub-standard, doubtful, and loss). A typical internal rating system might include six pass grades plus the four criticized grades, while the most detailed system might include 18 or more separate pass grades.

For risk measurement purposes, a customer's internal credit rating is generally used as a summary statistic for that customer's probability of defaulting on its credit obligations. This correspondence typically is accomplished through a two-step process which begins with the construction of a concordance table relating the bank's internal credit grades to some external standard, usually S&P's or Moody's ratings for corporate bonds. A grade-1 loan may be deemed roughly equivalent to an S&P bond rating from AA to AAA, a grade-2 loan equivalent to a bond rating of single-A, and so on. Given this concordance, the probability of a customer defaulting on its obligations over some horizon (or migrating to another credit risk grade) is usually inferred from published data on the historical credit rating migrations of similarly-rated corporate bonds.

In general, the process of arriving at a credit rating for a customer or facility can be described as containing one or more of the following three elements: 1) the traditional spreading of numbers, in which financial and other characteristics of the customer (e.g., country and SIC code) and specific features of the facility are incorporated into a relatively subjective approach to determining grades; 2) the use of vendor-supplied commercial credit scoring models; or 3) the use of internally-developed credit scoring models. To ensure consistency and discipline within the rating process, internal credit grades usually are reviewed by units within the bank that are independent of the businesses generating the credit exposures.

Internal credit ratings play an important role not only as a first step in the credit risk measurement process, but also as an important stand-alone risk management tool. Credit ratings are a basis for regular risk reports to senior management and boards of directors. They are also the basis for continuous loan review processes, under which large credits generally are reviewed and regraded at least annually in order to focus attention on deteriorating credits well before they become criticized by external auditors or examiners.

Definition of Credit Losses

Credit risk modeling procedures are shaped importantly by a bank's underlying definition of credit losses and the planning horizon over which such losses are measured. Banks generally employ a one-year planning horizon for analysis and what we shall refer to as either a default-mode (DM) or a mark-to-market (MTM) paradigm for defining and measuring credit losses. As described below, the DM paradigm embraces the notion that credit losses can arise only if a loan defaults during the planning horizon, while the MTM paradigm adopts the broader economic perspective that credit events short of default may generate declines, or increases, in economic value.

Default-Mode Paradigm. At present, the DM paradigm is the most common approach used by banks for defining credit losses. It is sometimes called a two-state model because only two outcomes are relevant: non-default and default. If the loan does not default, its future value (at the end of the planning horizon) is assumed to equal its current book

value; if the loan defaults, its future value is assumed to equal the present value of its expected net cash flows (e.g., recoveries less workout costs).¹¹

The DM paradigm can be thought of as a representation of the traditional buy-and-hold lending business of commercial banks. Under this view, secondary loan markets are not sufficiently developed to support a full mark-to-market or trading approach to risk measurement. Note that if all credit exposures had a one-year maturity (equal to the planning horizon), the DM paradigm could, at least conceptually, account for all potential credit losses within the portfolio. For instruments having an effective maturity exceeding one year, however, the DM framework largely ignores potential losses associated with defaults beyond the planning horizon.

Mark-To-Market Paradigm. The MTM paradigm, sometimes termed the multi-state model, generalizes the DM approach by recognizing that the economic value of a credit instrument may decline even if the counterparty does not formally default. Although few banks currently use the MTM framework outside their trading accounts, many practitioners believe the industry is likely to evolve from largely DM-based risk models for the banking book to the more general MTM-based models over the coming years. The MTM model is multi-state in that “default” is only one of several possible credit rating grades to which the instrument could migrate over the planning horizon.¹² In effect, the credit portfolio is assumed to be marked to market. A credit loss is defined as an unexpected reduction in the portfolio’s value over the planning horizon due to either deteriorations in credit ratings on the underlying loans or a widening of credit risk spreads in financial markets.¹³

To illustrate, consider a current grade-4 loan. Under both the DM and MTM paradigms, the loan could lose value if it were to default during the planning horizon. Under the MTM paradigm, however, credit losses also could arise if the loan were to suffer a downgrade short of default (e.g., move from grade 4 to grade 5), or if credit risk spreads on grade-4 loans were to increase over the planning horizon.

Valuation of Loans

The remainder of this section reviews the core analytical components of credit risk models. For each component, we first discuss its role and specification within the more general MTM framework, and then develop its DM counterpart as a special case.

Within both DM- and MTM-type credit risk models, the model-builder is required to specify precisely how the current and future values of each credit instrument are determined at the beginning and end of the planning horizon, respectively. To simplify the following exposition, we assume that a bank’s credit portfolio consists only of fixed-rate, term loans, and that each customer has only a single loan. (The Appendix presents a more formal mathematical description of the structure of the MTM model.)

¹¹ Precisely what constitutes a default varies somewhat across banks. Normally, a default arises if the obligor becomes unable to meet its payment obligations and is placed on nonaccruing status.

¹² The current state-of-the-art in MTM modeling is reflected, for example, in J.P. Morgan’s CreditMetrics™, which was released initially to clients in April 1997.

¹³ Unlike the DM model, within the MTM framework, the overall portfolio’s credit loss is not a simple summation of the losses across the individual assets. Portfolio losses almost always will be less than this sum, owing to diversification effects, as economic gains on some instruments due to favorable credit-related events (e.g., a rating upgrade) normally will offset at least some credit-related losses on other instruments.

Current Values of Loans. The current value of a loan typically is represented as the present discounted value of its *contractual* cash flows. The interest rates used for discounting contractual cash flows reflect: 1) the term structure of risk-free interest rates implied by the Treasury yield curve, and 2) for the obligors of each credit rating category, the term structure of credit risk spreads. This specification assumes that, apart from idiosyncratic random effects that wash out in the aggregate, credit risk spreads depend only on an instrument's credit rating (i.e., its probability of default).

Future Values of Non-Defaulted Loans. Consistent with the determination of current values, the future value of a non-defaulting loan (at the end of the planning horizon) would be calculated as the present discounted value of its remaining contractual cash flows.¹⁴ The discount rates can be different from those used in computing the loan's value at the beginning of the planning horizon, either because the loan's credit rating may have changed or because the term structure of credit spreads on loans of a given rating may have changed.

Future Values of Defaulted Loans. One of the rating grades to which a loan can migrate over the planning horizon is "default." Obviously, banks do not rely on the discounting of *contractual* cash flows for modeling the present values of *defaulted* loans. Rather, the decline in the economic value of a defaulted loan is typically represented in terms of the loan's loss-rate-given-default (LGD)—where LGD corresponds to the present value of the difference between the loan's contractual and actual net cash flows, per dollar of initial value. Usually, LGDs are assumed to depend on an instrument's seniority and collateral type *plus* a random risk factor.

Note that under the DM paradigm, only two future scenarios are relevant for valuing a loan at the end of the planning horizon: default and non-default. The future value of a non-defaulting loan is taken to be its book amount, while the decline in value of a defaulting loan is given by the loan's book value times its LGD (as is generally the case in MTM models).

Treatment of Credit-Related Optionality

In contrast to simple loans, for many types of credit instruments, a bank's credit exposure is not known with certainty, but rather may depend on the occurrence of future (random) events over which the customer may exercise some influence. One example of such credit-related optionality is a committed line of credit where, for a fixed period of time, a bank agrees to advance funds (up to a pre-defined credit limit) at the customer's discretion.¹⁵ A general characteristic of such lines is that customer draw-downs (per dollar of credit limit) tend to increase as a customer's credit quality deteriorates, reflecting the reduced availability or higher costs of alternative sources of funding.

Under the MTM framework, the credit-related optionality associated with a line of credit typically is modeled by treating the amount of draw-down over the planning horizon

¹⁴ For simplicity, principal and interest payments received during the planning horizon are assumed to be invested in (non-interest bearing) cash.

¹⁵ Credit-related optionality also arises with products outside a bank's traditional lending operations. For example, in the case of a derivative transaction, a bank's counterparty credit risk generally will vary randomly over the life of the contract, reflecting changes in the amount by which the bank is in-the-money.

(relative to the size of the undrawn commitment) as a known function of the customer's end-of-period credit rating. To illustrate, consider a one-year line of credit that is, initially, completely undrawn. Conditional on the customer's credit grade at the end of the planning horizon, the assumed draw-down under the line (per dollar of credit limit) might be based on the average historical draw-down experience of customers having that future grade. The future value of the line (conditional on the end-of-period credit grade) would then be calculated as if the line were a loan, the principal of which equaled the assumed draw-down for that grade.

Within the DM paradigm—as only two future credit ratings are relevant, default and non-default—a somewhat simpler approach is possible. In effect, the undrawn credit facility is converted into a Loan Equivalent Exposure (LEQ) to make it comparable to a term loan. Ideally, the LEQ would be calculated as the expected draw-down under the line in the event the customer were to become insolvent by the end of the period.¹⁶ (If the customer remains solvent, the size of the draw-down is irrelevant, as credit losses would equal zero.)

Specification and Estimation of Model Parameters

Under the MTM framework, three types of credit events can potentially lead to a change in the value of a loan: 1) a change in the loan's internal credit rating; 2) changes in credit spreads prevailing in financial markets; and 3) in the event of default, payments by the customer which may be less than its contractual obligation, represented by the LGD.¹⁷ In the most general case, each type of credit event would be modeled as an uncertain outcome driven by a random risk factor. (The Appendix presents a more formal characterization of these risk factors.)

The following discussion highlights what is perhaps the most challenging aspect of the credit risk modeling process, namely, the task of specifying the joint probability distribution of these risk factors. As will be noted repeatedly, data limitations normally render these estimates difficult and imprecise. Reflecting the longer-term nature of credit cycles, even in the best of circumstances (e.g., stable parameter values), many years of data, spanning multiple credit cycles, are needed to estimate the joint probability distribution with reasonable precision. At most banks, however, data on the historical performance of different types of loans have been warehoused only since the implementation of their capital allocation systems, often within the last few years. Thus, the model estimation process tends to be highly judgmental and to involve many crucial simplifying assumptions.

From standard portfolio theory, the overall uncertainty around a portfolio's rate of return depends on its systematic risk—that is, co-movements in loan values arising from their dependence on common influences. Within the MTM framework described above, four types of correlations among risk factors potentially could contribute to co-movements in loan valuations: 1) correlations between risk factors affecting credit-rating migrations, especially those corresponding to borrowers operating in related markets, such as the same geographic region or industrial sector; 2) correlations between risk factors determining LGDs; 3) correlations between risk factors driving changes in the term structures of credit

¹⁶ For a plain-vanilla term loan, the LEQ would equal the amount of the loan.

¹⁷ Changes in the risk-free yield curve are not treated as credit events.

risk spreads; and 4) cross-correlations among the risk factors affecting rating migrations, credit spreads, and LGDs.¹⁸ (Under the DM approach, of course, only three types of correlations are relevant—correlations between borrower defaults, correlations between LGDs, and cross-correlations among defaults and LGDs.)

Although critically important, correlations among random variables are difficult to estimate reliably with relatively short sample periods. Model-builders, therefore, tend to impose fairly restrictive assumptions on the pattern of correlations among the risk factors. In particular, credit risk models nearly always assume *zero* correlations between risk factors of different types. That is, the risk factors affecting changes in credit ratings are assumed to be independent of those affecting changes in risk premiums, which, in turn, are assumed to be independent of those affecting LGDs.

Risk Factors Affecting Loss Rates Given Default. Within the current generation of credit risk models, LGDs are usually assumed to be independently and identically distributed across all borrowers—and in some models, even across obligations of the same borrower—after controlling for a loan’s seniority or collateral type. The underlying probability distribution typically is inferred judgmentally from internal data on the bank’s own historical loan losses, consultants’ proprietary data on the LGDs of their clients, and/or historical LGD data culled from published articles on the subject.

Risk Factors Affecting Changes in Credit Risk Spreads. This area appears to be still in an early state of development, perhaps reflecting a lack of extensive databases on secondary market yields for lower-rated loans and bonds.¹⁹ (Under the DM paradigm, of course, changes in credit risk spreads are irrelevant.) For banks that have assembled such historical data, non-parametric approaches are sometimes used to estimate the joint probability distribution of future changes in credit risk spreads. One such procedure involves constructing, for each credit rating grade, a database of historical term structures of credit risk spreads. The joint probability distribution of future credit spreads is then estimated using a within-sample Monte Carlo simulation procedure.²⁰

Risk Factors Affecting Changes in Credit Ratings. Within most credit risk models, each customer’s credit rating at the end of the planning horizon is represented in terms of the future realization of a *migration risk factor* (i.e., an unobservable latent variable). The value of that customer-specific migration risk factor in relation to various thresholds is assumed to determine the change in that customer’s credit rating over the planning horizon. For example, given a customer’s current credit rating (say, BBB), an extremely large positive realization of the migration risk factor might correspond to an upgrade to AAA, a somewhat smaller (but still very large) positive value might correspond to an upgrade to AA, and so on. Similarly, an extremely large negative realization might

¹⁸ Apart from these correlations, a change in any credit spread would produce co-movements in loan values, as the same credit spreads are used in valuing *all* loans having a particular credit grade.

¹⁹ Within the current (April 1997) version of CreditMetrics™, for instance, the risk factors affecting changes in credit risk spreads actually are set to *zero* for purposes of modeling future loan values.

²⁰ That is, for each Monte Carlo iteration of the credit risk model, a separate historical date is chosen randomly. All the credit risk term structures used in computing future values for that iteration are then set equal to their levels as of that historical date. At the next iteration, another date is randomly selected, and the process repeated.

Exhibit 3. Sample Credit Rating Transition Matrix
(probability of migrating to another rating within one year, percent)

		Credit Rating One-Year in the Future							
		AAA	AA	A	BBB	BB	B	CCC	Default
Current credit rating	AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
	AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
	A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
	BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
	BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
	B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
	CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60

Source: *CreditMetrics Technical Document*, J.P. Morgan, April 2, 1997, p. 76.

Note: The credit rating transition matrix is based on the historical migration frequencies of publicly-rated corporate bonds. The transition probabilities in the table have been statistically smoothed in order to attenuate the effects of sampling variation in the actual migration patterns of corporate bonds.

generate a downgrade to default, etc. Primarily for analytical convenience, migration risk factors are often assumed to be jointly normally distributed.

MEANS AND VARIANCES OF MIGRATION RISK FACTORS. The stochastic properties of the migration risk factor associated with a particular borrower typically is represented through a ratings transition matrix, similar to that depicted in Exhibit 3. Given the customer’s current credit rating (delineated by each row), the probability of migrating to another grade (delineated by the columns) is shown within the intersecting cell. Thus, in the exhibit, the likelihood of a BBB-rated loan migrating to single-B within one year would be 0.32%.

The means and variances of the individual migration risk factors, together with the thresholds defining upgrades and downgrades, generally are reverse-engineered from the assumed credit rating transition matrix. Under the standard assumption that migration risk factors are normally distributed, this process is greatly simplified by virtue of the fact that, without loss of generality, the means and variances of the migration risk factors can be set to zero and unity, respectively. Thus, only the thresholds for rating migrations need to be estimated explicitly.²¹

In theory, one would expect the rating transition probabilities applicable to a given customer at a point in time to be conditional on various firm-specific and macroeconomic variables, such as the cyclical volatility of the firm’s earnings (perhaps proxied by SIC code) or the current stage of the business cycle. In practice, however, there is generally insufficient data with which to estimate transition probabilities at such detail with reasonable precision. Thus, at most banks, the same rating transition matrix usually is applied to all borrowers, with no adjustment for business cycle effects. (As, under the DM model, only rating migrations into the default state lead to changes in the values of loans, only the last column of this matrix would be relevant.)

CORRELATIONS BETWEEN MIGRATION RISK FACTORS. With regard to the correlations between the migration risk factors affecting different customers, estimation procedures across the sampled banks are quite diverse, but, again, invariably require many restrictive and

²¹ See Gupton et al. (1997, p. 92).

simplifying assumptions. One common approach involves stratifying the credit portfolio into a relatively small number of mutually exclusive sub-portfolios or buckets, for which annual historical default rates are available going back at least several years. Within each bucket, the loans are assumed to be statistically identical; that is, the correlation between any two risk factors is assumed to depend only on their respective buckets. Estimates of risk factor correlations for loans within the same, and across different, buckets are then inferred from the means, variances, and covariances of the historical default rates for the corresponding buckets.²²

For the most part, the stratification or bucketing schemes used in practice have tended to be based on internal credit ratings. However, the state-of-the-art in this area appears to be evolving rapidly toward more complex stratifications based on credit rating, industrial sector, *and* country. Some practitioners have even begun estimating correlations between migration risk factors on a name-by-name basis, without any stratification whatsoever, using option-theoretic techniques. These developments have been spurred by recent methodological advances that, under certain assumptions, permit correlations between migration risk factors to be inferred from co-movements in firms' equity prices.²³

PDF Computation Engine

In practice, the above components are combined into an estimate of a portfolio's PDF, generally via Monte Carlo simulation or via approximations using a mean/variance methodology. The Monte Carlo simulation techniques employed in credit risk modeling are substantively identical to those used in trading account VAR systems, and are not discussed here. Relatively few banks, however, currently use Monte Carlo methods to estimate PDFs. The vast majority use mean/variance approximations, which are viewed as computationally less burdensome. The mean/variance approach is premised on the assumption that a portfolio's PDF can be reasonably approximated by the probability density function of a beta (or in some cases, normal) distribution having the same estimated mean and variance.

Under the DM paradigm, the mean/variance approach implies that the economic capital allocation against credit risk for an individual credit facility is set at some multiple of that instrument's contribution to the overall portfolio's standard deviation of credit losses. The standard deviation of credit losses for the overall portfolio (σ) admits the following decomposition in terms of the contributions from individual credit facilities:

$$\sigma = \sum_{i=1}^N \sigma_i \rho_i, \quad (1)$$

where σ_i denotes the (stand-alone) standard deviation of credit losses for the i^{th} facility, and ρ_i denotes the correlation between credit losses on the i^{th} facility (per dollar of book value) and those on the overall portfolio.

²² This procedure involves a two-stage process. In the first stage, the means, variances, and covariances cited in the text are used to estimate *default correlations* between loans of various types. (For loans within the same bucket, this technique is illustrated in Gupton et al. (1997, pp. 159–160)). In the second stage, correlations between *migration risk factors* are estimated from the *default correlations* generated in the first step. The relationship between default correlations and migration risk factor correlations was developed in Zhou (1997).

²³ These advances have been pioneered by KMV Corp., a risk management consulting firm.

Under the further assumptions that (1) the random risk factors affecting customer defaults and LGDs are independent of one another, and (2) LGDs are independent across borrowers, the stand-alone standard deviation of credit losses for the i^{th} facility can be expressed as

$$\sigma_i = LEQ_i \sqrt{P_i(1 - P_i) \bar{LGD}_i^2 + P_i VOL_i^2}, \quad (2)$$

where LEQ is the instrument's loan-equivalent-exposure; P is the probability of default; \bar{LGD} is the expected loss-rate-given-default, and VOL is the standard deviation of the loss-rate-given-default.

These equations provide a convenient way of summarizing the overall portfolio's credit risk (within the DM framework) in terms of each credit exposure's P , ρ , \bar{LGD} , VOL , and LEQ . They also serve to highlight those aspects of the estimation process which determine the overall reliability of a credit risk model: 1) the accuracy of the above parameter estimates as representations of the future; and 2) the validity of the underlying independence and distributional assumptions.

Capital Allocation Rule

Once credit risk is estimated, the bank must invoke a particular rule for determining how much economic capital it should hold against this risk. As indicated above, at most institutions this capital allocation rule is expressed as the capital necessary to achieve some target insolvency rate over the planning horizon. For example, at some banks, this target is set around 0.03%, the historical default rate on AA-rated corporate bonds.

In cases where the PDF is estimated directly via Monte Carlo simulation, the economic capital allocation is computed directly from the estimated PDF, as shown in Exhibit 1. For banks using mean/variance approximation methods, economic capital is generally calculated as some multiple of the estimated standard deviation of portfolio credit losses. In practice, these multiples can vary widely (for example, between three and seven), depending on the target insolvency rate and on whether the true PDF is assumed to be normal- or beta-shaped. Final economic capital allocations, therefore, can differ considerably across banks, owing to differences in their respective capital allocation rules.

Another noteworthy feature of banks' internal capital allocation processes is their tendency to estimate different types of risk (e.g., credit, operating, and market risks) more or less independently of one another. That is, separate estimates are made for each type of risk, against which economic capital is allocated. The total economic capital allocation for the bank as a whole is then computed as the summation of the allocations for each risk type. Banks are aware that this piecemeal approach is not strictly consistent with their underlying portfolio framework—unless credit, market, and operating risks are perfectly correlated. Nevertheless, given the infeasibility of estimating inter-relationships (e.g., cross-correlations) among different types of risks, this approach is viewed as a practical necessity. Moreover, because the separate risks are certainly less than perfectly correlated in practice, the resultant capital allocations are generally believed to be conservative estimates of the overall capital needed to achieve the bank's target insolvency rate.²⁴

²⁴ Some banks allocate capital by estimating separate risk models for different lines of business (i.e., credit cards, other consumer lending, small business loans, and large corporate loans), and then allocating capital to each business on the basis of its stand-alone risk. This approach, too, is viewed as conservative.

V. An Internal Models Approach to Setting Formal RBC Requirements

In this section, we examine the near-term feasibility of replacing the Basle Accord with an internal models approach to setting regulatory capital requirements against credit risk in the banking book. The discussion focuses primarily on the extent to which the current state-of-the-art in credit risk modeling can be said to satisfy the qualitative standards for model integrity and reliability stipulated in the 1997 Market Risk Amendment. Specifically, these standards are interpreted as requiring that a risk measurement model be: 1) analytically sound; 2) subject to periodic backtesting and stress testing; and 3) well integrated into the bank's management decision making process.²⁵ We argue that the current generation of internal credit risk models raises important concerns in each of these dimensions.

Analytical Soundness

Within the current generation of credit risk models there are a number of important modeling issues:

Choice of Planning Horizon and Loss Paradigm. As noted above, banks typically employ a one-year planning horizon for purposes of credit risk modeling. This choice appears to be both pragmatic and, to some extent, arbitrary. On the one hand, in support of the one-year horizon, it is frequently suggested that this interval represents a reasonable period over which—in the normal course of business—a bank could mitigate its credit exposures (at least with respect to large corporate customers), taking into account improving liquidity in secondary loan markets and the average effective maturity of most credit instruments. The vast majority of commercial lines of credit, for example, tend to have maturities of one year or less. Another, and perhaps the most important consideration, is that the estimation of many key model parameters is often viewed as infeasible for planning horizons much beyond one year due to the lack of historical data.

On the other hand, from a supervisory perspective, the relevant issue is *not* the ease with which a *sound* bank could mitigate its exposures in the normal course of business. Rather, capital adequacy is normally considered within the context of a bank under *stress* attempting to unload the credit risk of a *significant* portfolio of *weak* credits. Whereas the markets for secondary loan trading and credit derivatives appear to be expanding and are becoming more liquid, they have not yet been tested by any large bank under severe stress. Indeed, the experience of the banking agencies suggests that several years is often required to resolve the portfolios of problem and/or failed banks and thrifts, over which time the deposit insurance funds remain exposed to potential further credit losses.

More generally, many credit instruments are subject to adverse selection which tends to increase the effective maturity of an instrument as its credit quality deteriorates. With regard to a nominal one-year loan commitment, for example, recall that banks typically experience greater rates of draw-down as a customer's credit rating declines. As a

²⁵ In addition to these model-specific requirements, the bank must have a risk control unit which reports directly to senior management and is independent from business units. The bank also must conduct independent reviews of its risk measurement and risk management policies and procedures. We do *not* discuss these requirements below because, with regard to credit risk models, formal compliance with these provisions is not likely to be a problem for any large U.S. bank having a comprehensive internal capital allocation system.

customer approaches insolvency, draw-downs under committed lines often approach 100%. When drawn, of course, the nominal one-year line of credit becomes effectively a loan, the scheduled maturity of which may extend another year or longer. Moreover, as a practical matter, the bank may continually extend or roll over the loan if it believes such actions will maximize its chances of recovery.

In light of such concerns regarding the actual liquidity of deteriorating portfolios in times of stress, supervisors may feel uncomfortable with the notion that capital is needed to cover only one year's worth of unexpected losses. Indeed, fluctuations in economic activity and in credit losses tend to be positively serially correlated from one year to the next, implying that a bank's capital buffer may be called upon to absorb significant credit losses extending beyond a single year.

Credit risk estimates generated under the DM paradigm may be particularly sensitive to the choice of a one-year planning horizon, because of the assumption that credit rating deteriorations short of default—even migrations to the worst non-default grade—have no adverse effect on a bank's financial condition. With respect to a three-year term loan, for example, this assumption could mean that more than two-thirds of the loan's credit risk is potentially ignored (corresponding to the possibility that the loan might default in the second or third years). Some banks, using DM models, have attempted to attenuate this concern by incorporating a facility's maturity into the determination of its credit risk grade or probability of default. That is, a shorter-term loan would tend to receive a better credit risk rating than a longer-term credit, other things being the same. It is difficult to assess the effectiveness of such adjustments given their *ad hoc* nature in many situations and the observation that similar adjustments for maturity generally are not made for other key model parameters (e.g., correlations).

Under the more general MTM framework, in contrast, it is assumed that the entire portfolio will be marked to market at the end of the planning horizon. Credit risk measurement under the MTM paradigm encompasses all potential reductions over the planning horizon in the portfolio's economic value due to credit quality deteriorations, whether to default or otherwise. Thus, capital allocations under the MTM framework may be less sensitive to the assumption of a one-year planning horizon than those calculated under the DM paradigm. At present, however, very few large banks use the MTM framework within their banking books, although some banks using the DM approach are considering switching to an MTM approach down the road.

LEQ and LGD Methodologies. Banks exhibit surprising diversity in their estimates of loan equivalent exposures (LEQs) for instruments containing credit-related optionality, such as credit lines. In some extreme cases, especially for lines functioning as credit enhancements for securitization programs, the LEQs computed by different banks may vary by as much as ten-fold for essentially similar facilities, reflecting differences in the sophistication of their underlying models for determining future draw-downs. No matter the process used for determining key parameters of its credit risk model, increasing (or decreasing) the LEQ for a particular instrument by a factor of X would increase (decrease) the resulting risk estimates and capital allocations for that instrument by the same factor.²⁶

²⁶ This can be seen directly from inspection of equations (1) and (2), above, where the contribution of a particular instrument to the portfolio's overall standard deviation of credit losses is directly proportional to its LEQ.

The sophistication of methods for estimating LGDs also varies considerably, especially for complex financial instruments supporting securitization activities. For example, it is not uncommon for banks to assume that, in the event of default, the LGD for a subordinated loan functioning as a credit enhancement for publicly-issued asset-backed securities would be comparable to the LGD of a corporate loan secured by similar assets (e.g., trade receivables or consumer credit). In the event of default, however, a \$25 million subordinated loan supporting a \$1 billion pool of securitized assets will tend to exhibit a much greater expected loss rate and loss rate volatility—corresponding to \bar{LGD} and VOL in equation (2)—than would a typical \$25 million senior corporate loan secured by similar assets. This is because the former will generally absorb a disproportionate share, in some cases (by design) essentially all, of the credit losses on the underlying asset pool. Given the growing importance of securitization, the risk exposures of some banks arising from credit enhancements may loom large in determining their overall capital adequacy.

Parameter Calibration. Under both the MTM and DM frameworks, estimates of portfolio credit risk are driven largely by assumptions and parameter estimates regarding the joint probability distribution of the relevant risk factors. Because available data on the historical performance of different types of loans generally do not span sufficiently long time periods (relative to the planning horizon) to enable precise estimation of this distribution, parameter values generally are established through a judgmental process involving considerable subjectivity and uncertainty.

To illustrate this process, consider that credit risk models often invoke simplifying assumptions such as the following: 1) joint normality of risk factors determining credit rating migrations, or, in the case of mean/variance DM models, the assumption that portfolio credit losses have a beta (or normal) probability distribution; 2) independence between risk factors affecting changes in credit ratings, changes in credit spreads, and LGDs; 3) independence of LGDs across borrowers; and 4) stability of model parameters.

In reviewing these assumptions, it should be noted that estimation of the extreme tail of a credit portfolio's PDF (the focus of credit risk models) is likely to be highly sensitive to variations in key parameters, such as correlations, or the assumption of joint normality. Typically, however, there is little analysis supporting the above assumptions; indeed, model-builders generally recognize that theoretical and empirical objections can be raised concerning their plausibility in many instances. Such assumptions generally are invoked for analytical convenience and to overcome data limitations which preclude the direct estimation of certain model parameters. Although such data problems do not preclude conducting sensitivity tests to gauge a credit risk model's vulnerability to key assumptions, surprisingly such tests are seldom conducted by the banks using these models. Similarly, when estimating credit risk, most practitioners assume that all parameters and assumptions are known with certainty, thus ignoring credit risk issues arising from uncertainty and/or instability in the model parameters.²⁷

To compensate for the lack of historical data on loan performance, model-builders have tended to assume that credit rating transition probabilities for large corporate credits, and correlations among the underlying risk factors, are identical to those for similarly-rated corporate bonds. With this assumption, model parameters generally are calibrated using public databases on the credit rating migrations of corporate bonds spanning twenty years

²⁷ Accounting for uncertainty in parameter estimates can significantly increase measured credit risk. See Duffee (1996).

or longer. Still, certain critical parameters (especially correlations among migration risk factors) often remain subject to large estimation uncertainty, requiring judgmental intervention by the model-builder. Given the degree of subjectivity in the specification of credit risk models, the need for effective model validation procedures is clearly paramount.

Model Validation: Backtesting and Stress Testing

In many ways, the task of estimating the extreme tail of the PDF is comparable to predicting the frequency at which credit losses in any year will exceed many multiples of a normal year's losses. This suggests that the only entirely objective method for evaluating the statistical accuracy of a credit risk model is to compare (over periods spanning multiple credit cycles) the model's *ex ante* estimates of PDFs against *ex post* realizations of actual credit losses. That is, only the realization of more frequent, extreme credit losses (relative to the model's predictions) can provide a purely statistical basis for concluding a model is deficient.²⁸ The detection of model shortcomings through standard out-of-sample backtesting methods, therefore, is almost certain to be extremely difficult in practice.²⁹ For this reason, *no* banks of which we are aware have yet developed formal backtesting procedures for their credit risk models.

In lieu of formal backtesting, credit risk models tend to be validated indirectly through various market-based reality checks. Peer group analysis, discussed in Section III, is used extensively to gauge the reasonableness of credit risk models and internal capital allocation processes. Another market-based validation technique involves comparing the bank's hurdle rate with the expected risk-adjusted rate of return (i.e., the RAROC) that could be achieved by investing in corporate bonds or syndicated loans having a particular credit rating, say, BB. An implied RAROC well below (above) the bank's hurdle rate might be interpreted as evidence that the model's capital allocation for BB-rated credits is too high (low), possibly requiring some recalibration of the model's parameters.

Clearly, an implicit assumption underlying these techniques is that market perceptions of appropriate capital levels or appropriate credit risk spreads are about right. If a bank elected to use such information to re-calibrate its risk model, re-calibration dates would need to be selected carefully so as to be reasonably confident that prevailing market perceptions were economically well founded. Otherwise, the bank could cede to the vagaries of the market much of the internal pricing and risk management discipline it had hoped to achieve through implementation of an economic capital allocation system. From a supervisory perspective, the use of market-based validation methods raises obvious concerns regarding the comparability and consistency of a risk model over time.

Lacking reliable backtesting procedures, it is difficult to envision how supervisors could objectively validate a bank's internal credit risk model. Although similar concerns were raised in the context of VAR models for market and specific risks within the *trading account*, this problem is much more acute in the context of internal risk models for the *banking book*. At most large banks, the size of the banking book and the length of its relevant planning horizon are much larger than those of the trading account, implying that

²⁸ Discussions of various statistical issues associated with validating such models are presented in Diebold et al. (1997).

²⁹ A similar concern has been raised in the context of the validation of trading account VAR models. See Kupiec (1995).

errors in measuring risks for the banking book are more likely to affect assessments of the institution's overall financial health. Moreover, the banking book does not benefit from relatively high liquidity and the discipline of a daily mark-to-market process which, in the context of the trading account, provide substantial safeguards against significant losses accumulating unnoticed and unaddressed.

As with trading account VAR models, appropriate stress testing of credit risk models for the banking book might partially compensate for shortcomings in backtesting. Most of the uncertainty within credit risk models (and the infeasibility of backtesting) relates to estimation of the joint probability distribution of risk factors. Stress tests circumvent these difficulties by specifying, albeit arbitrarily, particular economic scenarios against which the bank's capital adequacy might be judged—without regard to the probability of that event actually occurring. Stress testing is used routinely by the credit rating agencies, who often assign credit ratings on the basis of a security's ability to withstand various stress scenarios. Similarly, with respect to banks' trading activities, stress tests designed to simulate hypothetical market disturbances (e.g., the October 1987 stock market crash) provide useful checks on the reasonableness of the required capital levels generated by banks' VAR models. While, in principle, comparable stress tests might be developed for internal credit risk models used within the banking book, we are unaware of any efforts in this direction.

Integration of Credit Risk Models into Decision-Making Processes

The extent to which the output of a risk model is incorporated into a bank's actual decision-making processes is highly suggestive of management's confidence in that model. In practice, the extent of reliance on credit risk models differs greatly among banks. Much of this variation may reflect differences across institutions in the length of time over which their risk measurement and capital allocation systems have been operational. Generally speaking, the longer the period over which such systems have been in place, the greater their penetration into the bank's decision-making processes.

Within those institutions having the most sophisticated systems for allocating economic capital for credit risk, the outputs of these systems frequently are embedded throughout the bank's risk management and incentive systems. At such banks, economic capital allocations are critical components of the processes for determining breakeven prices on credit instruments, for setting customer credit limits and broad portfolio concentration limits, and, in some instances, for actively managing overall portfolio credit risk on a day-to-day basis. Moreover, at some institutions, risk-adjusted measures of profitability are significant factors in assessing customer profitability and managerial performance and compensation. Thus, among some banks, the commitment to the quantification of credit and other risks appears genuine.³⁰ Nevertheless, most internal capital allocation systems have been implemented only within the last five years, and the strengths of these commitments have not yet been tested under the stress of a full business cycle.

Although many banks use internal capital allocations for credit risk within a variety of decision-making environments, one aspect of these processes suggests an additional note

³⁰ Even within these institutions, however, there are notable instances where the output of credit risk models was not considered in situations where it might have been. In general, credit risk models are not used in determining loan loss reserve provisions. Neither are they used as a basis for fair value reporting and disclosures under FAS 125 and FAS 107, as amended.

of caution when contemplating the use of internal credit risk models for setting regulatory capital requirements. Credit and market risks are not the only types of risk that are measured, and against which economic capital is allocated. Besides credit and market risks, operating risks frequently account for a substantial fraction (25% or more) of large banks' total risk and allocated economic capital.³¹ Thus, assessing capital for credit and market risks, but not against operating risks, could substantially understate banks' overall capital needs. This problem is complicated by the fact that, while operating risks are viewed as quite important, models for quantifying these risks generally are primitive compared with those for market and credit risks. Before adopting an internal models approach to setting formal RBC requirements for *credit risk*, at a minimum, supervisors would need to consider carefully whether and how *operating risks* should be incorporated into the regulatory capital framework.

VI. Possible Near-Term Applications of Credit Risk Models

Although the reliability concerns raised above in connection with the *current* generation of credit risk models are substantial, they are not necessarily insurmountable. It may well be that credit and other risk models are evolving so rapidly that in the years ahead they may become the foundation for a new approach to setting formal regulatory capital requirements. Indeed, industry practitioners are now working on virtually all of the shortcomings described above. An advantage of waiting, moreover, is that the data-bases on loss experience will become more expansive and richer in detail. Whatever that time frame, it seems clear that if prudential capital policies are to remain an effective policy instrument even over the relatively near term, supervisors must begin to incorporate banks' internal risk measurement systems into their assessments of capital adequacy for the largest, most complex banking organizations (perhaps coupled with other initiatives to enhance market discipline and disclosure). Put simply, without appropriate analytical tools for quantifying credit risks, in practice, supervisors may have little objective basis for assessing capital adequacy until after credit quality or other problems have already surfaced and placed a bank under stress.

Given these concerns, within the relatively near term, there are at least two broad areas in which the inputs or outputs of banks' internal credit risk models might usefully be incorporated into prudential capital policies. These include: 1) the *selective* use of internal credit risk models in setting formal RBC requirements against certain credit positions which are not treated effectively within the current Basle Accord; and 2) the use of internal credit ratings and other components of credit risk models for purposes of developing specific and practicable *examination* guidance for assessing the capital adequacy of large, complex banking organizations.

Selective Use in Formal RBC Requirements

Under the current RBC standards, certain credit risk positions are treated ineffectually or, in some cases, ignored altogether. The selective application of internal risk models in this area could fill an important void in the current RBC framework for those instruments that,

³¹ Operating risk generally is defined rather broadly to encompass all risks that are not clearly credit risks or market risks.

by virtue of their being at the forefront of financial innovation, are the most difficult to address effectively through existing prudential techniques.

One particular application is suggested by the November 1997 Notice of Proposed Rulemaking on Recourse and Direct Credit Substitutes (NPR) put forth by the U.S. banking agencies. The NPR discusses numerous anomalies regarding the current RBC treatment of recourse and other credit enhancement supporting banks' securitization activities. In this area, the Basle Accord often produces dramatically divergent RBC requirements for essentially equivalent credit risks, depending on the specific contractual form through which the bank assumes those risks.

To address some of these inconsistencies, the NPR proposes setting RBC requirements for securitization-related credit enhancements on the basis of credit ratings for these positions obtained from one or more accredited rating agencies. One concern with this proposal is that it may be costly for banks to obtain formal credit ratings for credit enhancements which currently are not publicly rated. In addition, many large banks already produce internal credit ratings for such instruments which, given the quality of their internal control systems, may be at least as accurate as the ratings that would be produced by accredited rating agencies. A natural extension of the agencies' proposal would permit a bank to use its *internal* credit ratings (in lieu of having to obtain *external* ratings from accredited rating agencies) provided they were judged to be reliable by supervisors.

A further extension of the agencies' proposal might involve the direct use of internal credit risk models in setting formal RBC requirements for *selected* classes of securitization-related credit enhancements. Many current securitization structures were not contemplated when the Accord was drafted, and cannot be addressed effectively within the current RBC framework. Market acceptance of securitization programs, however, is based heavily on the ability of issuers to quantify (or place reasonable upper bounds on) the credit risks of the underlying pools of securitized assets. The application of internal credit risk models, if deemed reliable by supervisors, could provide the first practical means of assigning economically reasonable capital requirements against such instruments. The development of an internal models approach to RBC requirements—on a limited scale for selected instruments—also would provide a useful test-bed for enhancing supervisors' understanding and confidence in such models, and for considering possible expanded regulatory capital applications over time.³²

Improved Examination Guidance

As noted above, most large U.S. banks today have highly disciplined systems for grading the credit quality of individual financial instruments within major portions of their credit portfolios (e.g., large business customers). In combination with other information from banks' internal risk models, these internal grades could provide a basis for developing

³² As a referee pointed out, a *partial* models approach may be worse than the current Accord (if, for example, models are applied to generally low-risk activities, while high-risk activities continue to be subject to the one-size-fits-all Basle standard). Thus, a partial models approach might best be implemented in the context of a revised Accord that, for example, entailed a greater number of risk-buckets which more accurately reflected differences in risk across product types and/or internal rating categories. For a discussion of such a ratings-based revision to the Accord, see Mingo (1998).

specific and practical examination guidance to aid examiners in conducting independent assessments of the capital adequacy of large, complex banking organizations.

To give one example, in contrast to the one-size-fits-all Basle standard, a bank's internal capital allocation against a fully funded, unsecured commercial loan will generally vary with the loan's internal credit rating. Typical internal capital allocations often range from 1% or less for a grade-1 loan to 14% or more for a grade-6 loan (in a credit rating system with six pass grades). Internal economic capital allocations against classified, but not-yet-charged-off, loans may approach 40%—*not* counting any reserves for expected future charge-offs. Examiners could usefully compare a particular bank's actual capital levels (or its allocated capital levels) with the capital levels implied by such a grade-by-grade analysis (using as benchmarks the internal capital allocation ratios, by grade, of peer institutions). At a minimum, such a comparison could initiate discussions with the bank on the reliability of its internal approaches to risk measurement and capital allocation. Over time, examination guidance might evolve to encompass additional elements of banks' internal risk models, including analytical tools based on stress test methodologies.

Regardless of the specific details, the development and field testing of examination guidance on the use of internal credit risk models would provide useful insights into the longer-term feasibility of an internal models approach to setting regulatory capital standards. Although not committing supervisors in terms of how they might eventually use credit risk models for regulatory purposes, such an initiative would encourage further model development by banks, including greater efforts to resolve the reliability issues discussed above. The development of such guidance also would help ensure that supervisors remained abreast of ongoing developments in this field and were well-positioned to take advantage of future improvements in risk modeling practices.

To have a good chance of success, an examination-based internal models approach to assessing capital adequacy on a bank-by-bank basis would require that banks themselves have in place internal review processes for evaluating their own overall capital adequacy. Until such internal review processes are in place, it may be impractical for examiners to conduct independent assessments of capital adequacy and to engage senior management in constructive dialogues on this subject, absent clear indications of extant asset-quality problems. Arguably, internal reviews of capital adequacy should be a core element of any bank's overall risk management procedures and practices. However, often this is not the case, even at large banks which already have the basic components of such a process. To encourage greater progress in this direction, the banking agencies might consider issuing sound-practices guidance regarding the importance of such internal reviews, especially for the largest, most complex banks.³³

Concluding Remarks

The discussion above provides examples by which information from internal credit risk models might be usefully incorporated into regulatory or supervisory capital policies. In view of the modeling concerns described in Section V, incorporating internal credit risk measurement and capital allocation systems into the supervisory and/or regulatory frame-

³³ Recently, in the context of securitization and secondary loan market activities, the Federal Reserve issued similar guidance requiring that banks have in place appropriate internal risk measurement and capital allocation systems for managing the underlying credit and other risks [see Federal Reserve (1997)].

work will occur neither quickly nor without significant difficulties. Nevertheless, supervisors should not be dissuaded from embarking on such an endeavor. The current one-size-fits-all system of risk-based capital requirements increasingly is inadequate to the task of measuring large bank soundness. Moreover, the process of patching regulatory capital leaks as they occur appears to be less and less effective in dealing with the challenges posed by ongoing financial innovation and regulatory capital arbitrage. Finally, despite difficulties with an internal-models approach to bank capital, more attractive long-term solutions have not yet emerged.

The views expressed herein are those of the authors and do not necessarily reflect those of the Federal Reserve System or other members of its staff. This paper draws heavily upon information obtained through the authors' participation in an ongoing Federal Reserve System Task Force which has been reviewing the internal credit risk modeling and capital allocation processes of major U.S. banking organizations. A version of this paper was presented at the February 1998 Conference on Capital Regulation in the 21st Century (Federal Reserve Bank of New York) and reflects comments from other members of the System Task Force and Federal Reserve staff, including Raphael Bostic, Thomas Boemio, Roger Cole, Christine Cumming, Edward Ettin, Michael Gordy, Diana Hancock, Beverly Hirtle, James Hout, Myron Kwast, Mark Levonian, Chris Malloy, James Nelson, Thomas Oravez, Patrick Parkinson, and Thomas Williams. In addition, we have benefitted greatly from discussions with numerous practitioners in the risk management arena, especially John Drzik of Oliver, Wyman & Company. We also wish to thank two anonymous referees for helpful suggestions. We alone, of course, are responsible for any remaining errors.

References

- Aguais, S. D., Forest, L. Jr., Krishnamoorthy, S., and Mueller, T. Creating value from both loan structure and price. *Commercial Lending Review* (forthcoming).
- Altman, E. I., and Saunders, A. December 1997. Credit risk measurement: Developments over the last 20 years. *Journal of Banking and Finance* 21(11-12):1721-1742.
- Berger, A. N., Herring, R. J., and Szego, G. P. June 1995. The role of capital in financial institutions. *Journal of Banking and Finance* 19(3-4):393-430.
- Credit Suisse Financial Products. April 1997. *CreditRisk⁺: A Credit Risk Management Framework*.
- Diebold, F. X., Gunther, T. A., and Tay, A. S. Oct. 1997. Evaluating density forecasts. Technical Working Paper No. 215. National Bureau of Economic Research.
- Duffee, G. R. June 1996. On measuring credit risks of derivative instruments. *Journal of Banking and Finance* 20(5):805-833.
- Federal Reserve System. July 11, 1997. Risk management and capital adequacy of exposures arising from secondary market credit activities. Federal Reserve System, *SR Letter 97-12*.
- Federal Reserve System, Board of Governors. May 29, 1998. *Report of the System Task Force on Internal Credit Risk Models*.
- Gupton, G. M., Finger, C. C., and Bhatia, M. April 1997. *CreditMetricsTM—Technical Document*. New York: Morgan Guaranty Trust Co.
- Hendricks, D., and Hirtle, B. Sept. 22, 1997. Bank capital requirements for market risk: The internal models approach. Manuscript Federal Reserve Bank of New York.
- International Swaps and Derivatives Association. March 1998. Pamphlet *Credit Risk and Regulatory Capital*. ISDA. NY.
- Jones, D. S. Sept. 1998. Emerging problems with the Basle Accord: Regulatory capital arbitrage and related issues. Manuscript, delivered at a conference on *Credit Risk Modeling and the Regulatory Implications*. Bank of England.

- Jones, D., and King, K. K. June 1995. The implementation of prompt corrective action: An assessment. *Journal of Banking and Finance* 19(3-4):491-510.
- Kupiec, P. May 1995. Techniques for verifying the accuracy of risk measurement models. *Finance and Economics Discussion Series No. 95-24*. Board of Governors of the Federal Reserve System.
- Mingo, J. J. Sept. 1998. Policy implications of the Federal Reserve study of credit risk models at major U.S. banking institutions. Manuscript, delivered at a conference on *Credit Risk Modeling and the Regulatory Implications*. Bank of England.
- Robert Morris Associates. In conjunction with the First Manhattan Consulting Group. Sept. 1997. *Winning the Credit Cycle Game: A Road Map for Adding Shareholder Value through Credit Portfolio Management*.
- Wilson, T. Sept. 1997. Portfolio credit risk (I). *Risk Magazine* 10(9):111-117.
- Wilson, T. Oct. 1997. Portfolio credit risk (II). *Risk Magazine* 10(10):56-61.
- Zhou, C. May 1997. Default correlation: An analytical result. Mimeo. Board of Governors of the Federal Reserve System.

Appendix

The MTM Approach to Credit Risk Modeling

This appendix provides a more technical description of the stylized MTM credit risk model discussed in the text.

Valuation of Loans

Suppose the bank has N customers, where the current credit rating grade of the i^{th} customer is denoted g_i . The number of internal rating grades is denoted G , where grades 1 through $G-1$ are non-default states, and grade G represents a default. The term loan to the i^{th} customer has a contractual coupon payment of C_i dollars per period until maturity in period M_i , at which point the final payment (principal plus coupon) equals $C_i + P_i$.

Current Values of Loans. Given these assumptions, the current MTM value (at the beginning of the planning horizon) of a loan to the i^{th} customer equals the present discounted value of its contractual cash flows:

$$V_i = \frac{C_i}{[1 + {}_1r + {}_1R(g_i)]} + \frac{C_i}{[1 + {}_1r + {}_1R(g_i)][1 + {}_2r + {}_2R(g_i)]} + \cdots + \frac{C_i + P_i}{\prod_{k=1}^{M_i} [1 + {}_kr + {}_kR(g_i)]}. \quad (\text{A1})$$

The discount rate for period k equals the sum of: 1) the forward risk-free rate implied by Treasury term structure, denoted ${}_kr$; and 2) the market risk premium for deflating period- k contractual cash flows of g_i -rated obligors, denoted ${}_kR(g_i)$. In principle, the discount factors for the i^{th} customer could include a purely idiosyncratic component, affecting only that individual customer. However, to simplify the following exposition, this component is ignored. That is, credit risk spreads are assumed to depend only on the obligor/facility credit rating (g_i).

Future Values of Non-Defaulted Loans. From expression (A1), the value of a non-defaulting loan to the i^{th} customer as of the end of the planning horizon is given by:

$$V_i = C_i + \frac{C_i}{[1 + {}_2r + {}_2\hat{R}(\hat{g}_i)]} + \frac{C_i}{[1 + {}_2r + {}_2\hat{R}(\hat{g}_i)][1 + {}_3r + {}_3\hat{R}(\hat{g}_i)]} + \cdots + \frac{C_i + P_i}{\prod_{k=2}^{M_i-1} [1 + {}_kr + {}_k\hat{R}(\hat{g}_i)]}, \quad (\text{A2})$$

where a hat (^) or tilde (~) over a variable indicates that its value is taken as of the end of the planning period. A tilde signifies that the variable is exogenous (i.e., does not depend on other variables), while a hat signifies that it is endogenous. Thus, ${}_k\hat{R}(\hat{g}_i)$ denotes the market risk premium for obligors rated \hat{g}_i , where both the risk premium and the credit rating are endogenous variables measured as of the end of the holding period.

Future Values of Defaulted Loans. Banks generally do *not* rely on the valuation equation (A2)—which discounts *contractual* cash flows—for modeling the end-of-period values of *defaulted* loans. Whereas the default probability on a commercial loan might be reasonably expected to behave like that on the same obligor's bond, commercial loans tend to exhibit a very different seniority and collateral status. The decline in the economic value of a defaulted loan (relative to its book value, B_i) is typically determined as the loan's book value *times* its random loss rate given default ($L\hat{G}D$):

$$\hat{V}_i = B_i(1 - L\hat{G}D_i). \quad (\text{A3})$$

Within this simplified model, $L\hat{G}D$ s are assumed to equal the sum of a fixed average loss rate, L , and a zero-mean random error term, \tilde{l}_i :

$$L\hat{G}D_i = L + \tilde{l}_i. \quad (\text{A4})$$

Credit Rating Migrations

The likelihood of a facility migrating to another credit risk grade over the planning horizon is represented through a ratings transition matrix, similar to that depicted in Exhibit 3. For a given customer, a rating migration from g_i to \hat{g}_i is assumed to depend on the future realization of a customer-specific latent random variable, \tilde{v}_i , representing the change in that borrower's financial condition over the planning horizon.

Specifically, for an internal credit rating system with G grades:

$$\hat{g}_i = \begin{cases} 1 & \text{if } \tilde{v}_i \leq V_1(g_i) \\ 2 & \text{if } V_1(g_i) < \tilde{v}_i \leq V_2(g_i) \\ \vdots & \\ G-1 & \text{if } V_{G-1}(g_i) < \tilde{v}_i \leq V_G(g_i) \\ G & \text{Otherwise,} \end{cases} \quad (\text{A5})$$

where for a customer having a credit rating of g_i , the $V_1(g_i), \dots, V_G(g_i)$ denote the threshold levels of \tilde{v}_i that trigger rating downgrades or upgrades. Thus, for a grade-4 facility (i.e., $g_i = 4$) a value of \tilde{v}_i less than or equal to $V_1(4)$ would imply a future credit

rating of grade-1, a value greater than $V_1(4)$ but less than or equal to $V_2(4)$ would imply a grade-2, and so forth. Mathematically, the threshold levels are chosen so that the probability of any borrower migrating to another grade, given its current rating, agrees with the assumed rating transition matrix.

Changes in Credit Risk Spreads

It is assumed that for a given credit rating, g , changes in the credit risk spread for period k are random:

$${}_k\hat{R}(g) = {}_kR(g) + \tilde{z}_k(g), \quad \text{for } k = 1, 2, \dots, M, \quad (\text{A6})$$

where M is the longest maturity of any loan, and $\tilde{z}_k(g)$ denotes a random risk factor. (In practice, the model for credit risk spreads may be expressed in terms of relative or logarithmic changes in yields, rather than absolute changes in yields.)

Risk Factors

The main body of this paper refers to three types of risk factors within the MTM model. In terms of the above model specification, these risk factors correspond to: 1) the random variables affecting rating migrations (the \tilde{v}_i , for $i = 1, 2, \dots, N$); 2) the random variable affecting credit risk spreads (the $\tilde{z}_k(g)$), for $g = 1, 2, \dots, G$, and $k = 1, 2, 3, \dots, M$); and 3) the random variables affecting loss-rates-given-default (the \tilde{l}_i , for $i = 1, 2, \dots, N$).